1. **Multiple Choice** A plane intersects a right cone so that it is perpendicular to the axis of rotation of the cone. Which conic is formed?
   - A. circle
   - B. ellipse
   - C. hyperbola
   - D. parabola

2. **Explain how an ellipse is formed when a cone is intersected by a plane.**
   
   An ellipse is formed when a plane intersects a cone so that the angle between the plane and the rotational axis of the cone is less than 90°, but greater than the angle of the generator of the cone.
10.2

3. Multiple Choice Which equation describes a circle centred at (3, −2) and with radius 4?

A. \((x + 3)^2 + (y - 2)^2 = 4\)    \(\text{B. } (x - 3)^2 + (y + 2)^2 = 8\)
C. \((x + 3)^2 + (y - 2)^2 = 16\)    \(\text{D. } (x - 3)^2 + (y + 2)^2 = 16\)

4. a) Determine the equation of a circle centred at (3, −2) and passing through (6, −2).

The equation of the circle has the form: \((x - h)^2 + (y - k)^2 = r^2\)
Since the two given pairs of coordinates have the same \(y\)-coordinate, the radius of the circle is the difference in the \(x\)-coordinates, so:
\[ r = 6 - 3, \text{ or } 3 \]
In the equation above, substitute: \(h = 3, k = -2, \text{ and } r = 3\)
The equation of the circle is: \((x - 3)^2 + (y + 2)^2 = 9\)

b) Graph the circle and identify its domain and range.

From the graph:
the domain is: \(0 \leq x \leq 6\)
the range is: \(-5 \leq y \leq 1\)

c) Which transformation will move this circle so that it is centred at the origin?

A translation 3 units left and 2 units up will move the circle so that it is centred at the origin.

5. a) i) For the circle described by \((x + 8)^2 + (y - 3)^2 = 36\), determine the radius and the coordinates of its centre.

The radius, \(r = \sqrt{36}, \text{ or } 6\)
The coordinates of the centre are: \((-8, 3)\)
ii) Which transformation would result in an image circle, centred at the origin, and radius 1?

The given circle would be translated 8 units right and 3 units down, then compressed vertically and horizontally by a factor of \( \frac{1}{6} \).

b) i) Which transformation would move the circle described by 
\[(x + 8)^2 + (y - 3)^2 = 36\] so that the centre has coordinates 
\((-7, -7)\)?

The centre of the circle moves from point \((-8, 3)\) to \((-7, -7)\), so the translation is 1 unit right and 10 units down.

ii) What is the equation of this image circle?

The equation of the image circle is: 
\[(x + 7)^2 + (y + 7)^2 = 36\]

iii) What are the domain and range of this image circle?

The radius of the circle is 6.
The coordinates of the endpoints of the horizontal diameter are: 
\((-7 - 6, -7)\), or \((-13, -7)\); and 
\((-7 + 6, -7)\), or \((-1, -7)\)
The coordinates of the endpoints of the vertical diameter are: 
\((-7, -7 - 6)\), or \((-7, -13)\); and 
\((-7, -7 + 6)\), or \((-7, -1)\)
So, the domain is: 
\(-13 \leq x \leq -1\)
the range is: 
\(-13 \leq y \leq -1\)

c) The circle described by 
\[(x + 8)^2 + (y - 3)^2 = 36\] is reflected in the \(x\)-axis.

i) What is the equation of the image circle?

After the reflection, the centre of the image circle has coordinates: 
\((-8, -3)\)
Its radius does not change.
The equation of the image circle is: 
\[(x + 8)^2 + (y + 3)^2 = 36\]

ii) What are the domain and range of this image circle?

The radius of the circle is 6.
So, the coordinates of the endpoints of the horizontal diameter are: 
\((-8 - 6, -3)\), or \((-14, -3)\); and 
\((-8 + 6, -3)\), or \((-2, -3)\)
The coordinates of the endpoints of the vertical diameter are: 
\((-8, -3 - 6)\), or \((-8, -9)\); and 
\((-8, -3 + 6)\), or \((-8, 3)\)
So, the domain is: 
\(-14 \leq x \leq -2\)
the range is: 
\(-9 \leq y \leq 3\)
iii) Which transformation would map this image circle onto the image circle in part b?

The centre of this image circle has coordinates: (−8, −3)
The centre of the image circle in part b has coordinates (−7, −7).
So, the translation is 1 unit right and 4 units down.

6. For the circle with the equation $x^2 + y^2 + 4x - 6y - 12 = 0$, determine its radius and the coordinates of its centre.

Write the equation in standard form by completing the square for $x$ and for $y$.

$x^2 + y^2 + 4x - 6y - 12 = 0$

$(x^2 + 4x) + (y^2 - 6y) - 12 = 0$

$(x^2 + 4x + 4) + (y^2 - 6y + 9) - 12 = 0$

$(x + 2)^2 + (y - 3)^2 - 25 = 0$

$(x + 2)^2 + (y - 3)^2 = 25$

The coordinates of the centre are: (−2, 3)

$r^2 = 25$

$r = 5$

The radius is 5.

10.3

7. **Multiple Choice** What are the coordinates of the centre and the length of the major axis of the ellipse described by the equation

$$
\frac{(x - 3)^2}{25} + \frac{(y + 2)^2}{36} = 1?
$$

A. (3, −2); 12
B. (−3, 2); 10
C. (3, −2); 6
D. (3, −2); 5

8. An ellipse is described by the equation: $4(x - 4)^2 + 49(y + 6)^2 = 196$

a) Write the equation in standard form. Identify the coordinates of the centre of the ellipse.

$4(x - 4)^2 + 49(y + 6)^2 = 196$

Divide by 196.

$$
\frac{(x - 4)^2}{49} + \frac{(y + 6)^2}{4} = 1
$$

The coordinates of the centre are: (4, −6)
b) Determine the directions and lengths of the major and minor axes.

\[ a^2 = 49 \]
\[ a = 7 \]
\[ b^2 = 4 \]
\[ b = 2 \]

Since \( a > b \):
the major axis is horizontal, and its length is: \( 2a = 2(7) \), or 14
the minor axis is vertical, and its length is: \( 2b = 2(2) \), or 4

c) Graph the ellipse and identify its domain and range.

Label the coordinates of the endpoints of the major and minor axes.
the domain is: \(-3 \leq x \leq 11\)
the range is: \(-8 \leq y \leq -4\)

d) Which transformation(s) would result in an image that is an ellipse centred at the origin?

For an image ellipse with centre at \((0, 0)\), the given ellipse would be translated 4 units left and 6 units up.

9. The coordinates of the centre of an ellipse are \((6, -2)\).
The major axis is vertical and 14 units long. The minor axis is 10 units long.

a) Write the equation of the ellipse in standard form.

Use: \( \frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1 \)

\[ 2b = 14, \text{ so } b = 7 \text{ and } 2a = 10, \text{ so } a = 5 \]
Substitute these values for \( a, b, \) and \( h = 6 \) and \( k = -2 \).

\[ \frac{(x - 6)^2}{5^2} + \frac{(y - (-2))^2}{7^2} = 1 \]
\[ \frac{(x - 6)^2}{25} + \frac{(y + 2)^2}{49} = 1 \]
b) Which transformations would produce an image that is a circle with its centre at the origin and radius 1?

To produce an image circle, centred at the origin, the ellipse would be translated 6 units left and 2 units up, then compressed horizontally by a factor of \( \frac{1}{5} \) and compressed vertically by a factor of \( \frac{1}{7} \).

10. Consider the equation: \( 4x^2 + 9y^2 + 8x - 54y + 49 = 0 \)

a) How can you tell that this equation describes an ellipse?

When the equation of a conic is written in general form, it describes an ellipse if \( A \) and \( B \) have the same signs, and \( A \neq B \neq 0 \). In this equation, \( A = 4 \) and \( B = 9 \) so the equation describes an ellipse.

b) Write the equation in standard form.

\[
\begin{align*}
4x^2 + 9y^2 + 8x - 54y + 49 &= 0 \\
(4x^2 + 8x) + (9y^2 - 54y) + 49 &= 0 \\
4(x^2 + 2x) + 9(y^2 - 6y) + 49 &= 0 \\
4(x^2 + 2x + 1 - 1) + 9(y^2 - 6y + 9 - 9) + 49 &= 0 \\
4(x + 1)^2 + 9(y - 3)^2 &= 36 \\
\frac{4(x + 1)^2}{36} + \frac{9(y - 3)^2}{36} &= 1 \\
\frac{(x + 1)^2}{9} + \frac{(y - 3)^2}{4} &= 1
\end{align*}
\]

c) Identify the domain and range of the ellipse.

The domain is: \( -4 \leq x \leq 2 \)
The range is: \( 1 \leq y \leq 5 \)