3. Write an equation to describe each circle.

a)

The range is: $-6 \leq y \leq 6$
The diameter is 12 units, so the radius is 6.
The domain is: $-8 \leq x \leq 4$
The coordinates of the centre are: $(-2, 0)$
Use $(x - h)^2 + (y - k)^2 = r^2$ with the centre at $(-2, 0)$ and radius 6.
$(x - (-2))^2 + (y - 0)^2 = 6^2$
$(x + 2)^2 + y^2 = 36$

b)

The range is: $-2 \leq y \leq 6$
The diameter is: $6 - (-2) = 8$, so the radius is 4
The domain is: $1 \leq x \leq 9$
The coordinates of the centre are: $(5, 2)$
Use $(x - h)^2 + (y - k)^2 = r^2$ with the centre at $(5, 2)$ and radius 4.
$(x - 5)^2 + (y - 2)^2 = 4^2$
$(x - 5)^2 + (y - 2)^2 = 16$
4. For each circle, identify the radius and the coordinates of the centre.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
</table>
| a) \(x^2 + y^2 = 49\) | Use: \(x^2 + y^2 = r^2\)  
\(r^2 = 49\)  
\(r = \sqrt{49}, \text{ or } 7\)  
Centre: \((0, 0)\)  
| b) \((x - 4)^2 + (y + 1)^2 = 64\) | Use: \((x - h)^2 + (y - k)^2 = r^2\)  
\((x - 4)^2 + (y - (-1))^2 = 64\)  
\(r^2 = 64\)  
\(r = \sqrt{64}, \text{ or } 8\)  
Centre: \((4, -1)\)  
| c) \((x + 2)^2 + (y - 3)^2 = 40\) | Use: \((x - h)^2 + (y - k)^2 = r^2\)  
\((x - (-2))^2 + (y - 3)^2 = 40\)  
\(r^2 = 40\)  
\(r = \sqrt{40}, \text{ or } 2\sqrt{10}\)  
Centre: \((-2, 3)\)  

5. Each circle below is a transformation image of the circle described by the equation \(x^2 + y^2 = r^2\). For each circle below:

i) Identify the transformation.
ii) Write the equation in general form.

<table>
<thead>
<tr>
<th>Circle</th>
<th>Description</th>
</tr>
</thead>
</table>
| a) \((x + 7)^2 + y^2 = 36\) | i) The coordinates of the centre of the given circle are: \((0, 0)\)  
The coordinates of the centre of the image circle are: \((-7, 0)\)  
The transformation is a translation of 7 units left.  
ii) \((x^2 + 14x + 49) + y^2 - 36 = 0\)  
\(x^2 + y^2 + 14x + 49 - 36 = 0\)  
\(x^2 + y^2 + 14x - 13 = 0\)  
| b) \((x - 5)^2 + (y + 8)^2 = 121\) | i) The coordinates of the centre of the given circle are: \((0, 0)\)  
The coordinates of the centre of the image circle are: \((5, -8)\)  
The transformation is a translation of 5 units right and 8 units down.  
ii) \((x^2 - 10x + 25) + (y^2 + 16y + 64) - 121 = 0\)  
\(x^2 + y^2 - 10x + 16y + 25 + 64 - 121 = 0\)  
\(x^2 + y^2 - 10x + 16y - 32 = 0\)  

6. Each equation below represents the translation image of a circle that was centred at the origin.

For each translated circle:

i) Determine its radius and the coordinates of its centre.
ii) Identify the translation.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
</table>
| a) \(x^2 + y^2 + 6y - 7 = 0\) | i) Write the equation in standard form by completing the square for \(y\).  
\(x^2 + (y^2 + 6y) - 7 = 0\)  
\(x^2 + (y^2 + 6y + 9 - 9) - 7 = 0\)  
\(x^2 + (y^2 + 6y + 9) - 9 - 7 = 0\)  
\(x^2 + (y + 3)^2 - 16 = 0\)  
\(x^2 + (y + 3)^2 = 16\)  
The coordinates of the centre are \((0, -3)\).  
\(r^2 = 16\)  
\(r = \sqrt{16}, \text{ or } 4\)  
The radius is 4.  
ii) The translation is 3 units down.  

b) \(x^2 + y^2 - 4x + 10y + 28 = 0\)

i) Write the equation in standard form by completing the square for \(x\) and for \(y\).

\[
\begin{align*}
(x^2 - 4x) + (y^2 + 10y) + 28 &= 0 \\
(x^2 - 4x + 4) + (y^2 + 10y + 25) - 4 - 25 + 28 &= 0 \\
(x - 2)^2 + (y + 5)^2 - 1 &= 0 \\
(x - 2)^2 + (y + 5)^2 &= 1
\end{align*}
\]

The coordinates of the centre are \((2, -5)\).
\(r^2 = 1\)
\(r = \sqrt{1}, \text{ or } 1\)
The radius is 1.

ii) The translation is 2 units right and 5 units down.

c) \(x^2 + y^2 + 16x - 6y + 33 = 0\)

i) Write the equation in standard form by completing the square for \(x\) and for \(y\).

\[
\begin{align*}
(x^2 + 16x) + (y^2 - 6y) + 33 &= 0 \\
(x^2 + 16x + 64 - 64) + (y^2 - 6y + 9 - 9) + 33 &= 0 \\
(x^2 + 16x + 64) + (y^2 - 6y + 9) - 64 - 9 + 33 &= 0 \\
(x + 8)^2 + (y - 3)^2 - 10 &= 0 \\
(x + 8)^2 + (y - 3)^2 &= 40
\end{align*}
\]

The coordinates of the centre are \((-8, 3)\).
\(r^2 = 40\)
\(r = \sqrt{40}, \text{ or } 2\sqrt{10}\)
The radius is 2\(\sqrt{10}\).

ii) The translation is 8 units left and 3 units up.

7. Suppose a circle is reflected in the \(y\)-axis. How does the graph of the circle change in each case? Justify your answers.

a) The centre of the original circle is on the \(y\)-axis.

Reflecting the circle in the \(y\)-axis will not affect the graph of the circle.

b) The centre of the original circle is on the \(x\)-axis.

If the coordinates of the centre of the original circle are \((h, 0)\), the centre of the image circle will have coordinates \((-h, 0)\).

c) The centre of the original circle is not on an axis.

If the coordinates of the centre of the original circle are \((h, k)\), the centre of the image circle will have coordinates \((-h, k)\).
8. A circle described by the equation \( x^2 + y^2 = 25 \) is translated 2 units left and 6 units down.

a) What is the equation of the image circle?

The coordinates of the centre of the given circle are: (0, 0)
The coordinates of the centre of the image circle are: (−2, −6)
So, the equation of the image circle is: \((x + 2)^2 + (y + 6)^2 = 25\)

b) Sketch the image circle and determine its domain and range.

From the coordinates of the endpoints of the horizontal and vertical diameters:
the domain is: \(-7 \leq x \leq 3\)
the range is: \(-11 \leq y \leq −1\)

9. A circle is described by the equation \( x^2 + y^2 − 4x + 12y = 0 \).

a) Which transformation will move the circle so that its centre is at the origin?

Write the equation in standard form.
\[
\begin{align*}
x^2 + y^2 - 4x + 12y & = 0 \\
(x^2 - 4x) + (y^2 + 12y) & = 0 \\
(x^2 - 4x + 4 - 4) + (y^2 + 12y + 36 - 36) & = 0 \\
(x^2 - 4x + 4) + (y^2 + 12y + 36) - 4 - 36 = 0 \\
(x - 2)^2 + (y + 6)^2 & = 40 \\
\end{align*}
\]
The coordinates of the centre of the given circle are: (2, −6)
The coordinates of the centre of the image circle are: (0, 0)
So, the transformation is a translation 2 units left and 6 units up.

b) What is the equation of the image circle?

The equation of the image circle is: \( x^2 + y^2 = 40 \)
10. a) Write the equation $4x^2 + 4y^2 - 12x + 2y + 7 = 0$ in standard form. Identify the radius and the coordinates of the centre of the circle.

\[
4x^2 + 4y^2 - 12x + 2y + 7 = 0 \quad \text{Divide each term by 4.}
\]

\[
x^2 + y^2 - 3x + \frac{1}{2}y + \frac{7}{4} = 0
\]

\[
\left( x^2 - 3x + \frac{9}{4} \right) + \left( y^2 + \frac{1}{2}y + \frac{1}{16} \right) + \frac{7}{4} = 0
\]

\[
\left( x^2 - 3x + \frac{9}{4} \right) + \left( y^2 + \frac{1}{2}y + \frac{1}{16} \right) - \frac{9}{4} - \frac{1}{16} + \frac{7}{4} = 0
\]

\[
\left( x - \frac{3}{2} \right)^2 + \left( y + \frac{1}{4} \right)^2 = \frac{9}{16}
\]

The coordinates of the centre are: \(\left( \frac{3}{2}, -\frac{1}{4} \right)\)

\[r^2 = \frac{9}{16}\]

\[r = \sqrt{\frac{9}{16}}\]

\[r = \frac{3}{4}\]

The radius is \(\frac{3}{4}\).

b) Which translation would move the circle in part a so that its centre is at the point (5, 2)?

The coordinates of the centre of the given circle are: \(\left( \frac{3}{2}, -\frac{1}{4} \right)\)

The coordinates of the centre of the image circle are: (5, 2)

The circle has to move:

\[
5 - \frac{3}{2} = \frac{7}{2} \quad \text{to the right}
\]

The circle has to move:

\[
2 - \left( -\frac{1}{4} \right) = \frac{9}{4} \quad \text{up}
\]

So, the translation is \(\frac{7}{2}\) units right and \(\frac{9}{4}\) units up.
11. For each circle:
   
i) Determine the radius and the coordinates of its centre.
   
   ii) Sketch the circle and identify its domain and range.
   
   iii) Which transformation(s) would result in an image circle with
         its centre at the origin and radius 1?
   
   a) \((x - 8)^2 + (y + 6)^2 = 100\)
      
i) The radius, \(r = \sqrt{100}\), or 10
      
The coordinates of the centre are: \((8, -6)\)
      
      ii) [Diagram]
      
      iii) For an image circle with centre
           
           \((0, 0)\), and radius 1, the equation is:
           
           \(x^2 + y^2 = 1\)
           
           The given circle would be translated
           
           8 units left and 6 units up, then
           
           compressed horizontally and
           
           vertically by a factor of \(\frac{1}{10}\).
           
      The domain is: \(-2 \leq x \leq 18\)
      
      The range is: \(-16 \leq y \leq 4\)
      
   b) \((x - 5)^2 + (y - 2)^2 = 16\)
      
i) The radius, \(r = \sqrt{16}\), or 4
      
The coordinates of the centre are: \((5, 2)\)
      
      ii) [Diagram]
      
      iii) For an image circle with centre
           
           \((0, 0)\), and radius 1, the equation is:
           
           \(x^2 + y^2 = 1\)
           
           The given circle would be translated
           
           5 units left and 2 units down, then
           
           compressed horizontally and
           
           vertically by a factor of \(\frac{1}{4}\).
           
      The domain is: \(1 \leq x \leq 9\)
      
      The range is: \(-2 \leq y \leq 6\)
12. For what value of \( k \) will the point \((4, -6)\) lie on the circle defined by \(x^2 + y^2 - 10x - 4y + k = 0\)?

In \(x^2 + y^2 - 10x - 4y + k = 0\), substitute \(x = 4\) and \(y = -6\), then solve for \(k\).

\[
4^2 + (-6)^2 - 10(4) - 4(-6) + k = 0
\]
\[
36 + k = 0
\]
\[
k = -36
\]

13. A circle has a diameter PQ with endpoints P\((-5, -7)\) and Q\((3, -1)\).
Write the equation of the circle in standard form.

Sketch the circle.

![Diagram of a circle with points P and Q]

To determine the diameter, \(d\), use the distance formula with the coordinates of P and Q.

\[
d = \sqrt{(-5 - 3)^2 + (-7 + 1)^2}
\]
\[
d = \sqrt{100}
\]
\[
d = 10
\]

So, the radius is 5.
The diameter passes through the centre of the circle.

\(x\)-coordinate of the centre:

\[
\frac{-5 + 3}{2}, \text{ or } -1
\]

\(y\)-coordinate of the centre:

\[
\frac{-7 + 1}{2}, \text{ or } -4
\]

The coordinates of the centre are: \((-1, -4)\)

Use:

\[
(x - h)^2 + (y - k)^2 = r^2
\]

\[
(x - (-1))^2 + (y - (-4))^2 = 5^2
\]

\[
(x + 1)^2 + (y + 4)^2 = 25
\]

14. Is it correct to say that every circle is its own inverse? Justify your answer.

An inverse is formed by interchanging the \(x\)- and \(y\)-coordinates; that is, the relation is reflected in the line \(y = x\).

If the coordinates of the centre of the circle are \((0, 0)\), the circle is its own inverse.

A circle will also be its own inverse if its centre lies on the line \(y = x\).

All other circles are not their own inverses.

15. Is it possible to stretch or compress a circle horizontally or vertically and still have it remain a circle? Justify your answer.

If a circle is stretched or compressed horizontally and vertically by the same amount, the circle will remain a circle. Consider the circle \(x^2 + y^2 = 16\).
Suppose it is compressed both horizontally and vertically by a factor of \(\frac{1}{2}\).
The equation of the image is:

\[
(2x)^2 + (2y)^2 = 16
\]
\[
4x^2 + 4y^2 - 16 = 0
\]

In general form, \(A = B = \frac{1}{4}\) so the conic is still a circle.