Lesson 9.1 Exercises, pages 624–628

Exercises

Use graphing technology to solve each equation. Where necessary, round the roots to the nearest hundredth.

A

4. To solve the equation $3 \cos x = 1.5$ for $0 \leq x \leq 2\pi$,
   enter the equations: $y = 3 \cos x$ and $y = 1.5$
   What are the roots of the equation $3 \cos x = 1.5$?

   I graphed the function, then determined the approximate $x$-coordinate of each point of intersection.
   To the nearest hundredth, the roots are: $x = \pm 5.24, x = \pm 1.05$

5. Solve each equation for $0 \leq x < 2\pi$.
   a) $\sin x = \frac{2}{5}$
      Graph $y = \sin x$ and $y = \frac{2}{5}$.
      To the nearest hundredth, the roots are: $x = 0.41$ and $x = 2.73$
   b) $\cos x = -\frac{1}{3}$
      Graph $y = \cos x$ and $y = -\frac{1}{3}$.
      To the nearest hundredth, the roots are: $x = 1.91$ and $x = 4.37$
6. Solve the equation \( \sin x = -\frac{4}{7} \) over the domain \( 0 \leq x < 2\pi \).
Assume \( x \) is an angle in standard position. In which quadrants do the terminal arms of the angles lie? How do you know?

Graph \( y = \sin x \) and \( y = -\frac{4}{7} \)
To the nearest hundredth, the roots are: \( x = 3.75 \) and \( x = 5.67 \)
The terminal arms lie in Quadrants 3 and 4 because the sine of an angle is negative when its terminal arm lies in those quadrants.

7. Solve each equation for \( -2\pi \leq x < 0 \).

a) \( \tan x - 3 = \cos x + 2 \)
Graph \( y = \tan x - 3 \) and \( y = \cos x + 2 \).
To the nearest hundredth, the roots are: \( x = -4.90 \) and \( x = -1.78 \)

b) \( 2 = 4 \sin x - 3 \cos x \)
Graph \( y = 2 \) and \( y = 4 \sin x - 3 \cos x \).
To the nearest hundredth, the roots are: \( x = -5.23 \) and \( x = -2.91 \)

8. To solve the equation \( \cos^2 x = \tan x \) for \( 0 \leq x \leq 2\pi \), enter the equations: \( y = \cos^2 x \) and \( y = \tan x \)
What are the roots of the equation \( \cos^2 x = \tan x \)?
To the nearest hundredth, the roots are: \( x = 0.60 \) and \( x = 3.74 \)

9. Solve each equation for \( 0 \leq x < 2\pi \), then write the general solution.

a) \( 5 \sin^2 x - \sin x = 2 \)
Graph \( y = 5 \sin^2 x - \sin x - 2 \).
To the nearest hundredth, the roots are: \( x = 0.83, x = 2.31, x = 3.71, \) and \( x = 5.71 \)
The period is \( 2\pi \), so the general solution is approximately:
\( x = 0.83 + 2\pi k, k \in \mathbb{Z} \) or \( x = 2.31 + 2\pi k, k \in \mathbb{Z} \) or
\( x = 3.71 + 2\pi k, k \in \mathbb{Z} \) or \( x = 5.71 + 2\pi k, k \in \mathbb{Z} \)

b) \( 3 \tan x - 1 = \tan^2 x \)
Graph \( y = \tan^2 x - 3 \tan x + 1 \).
To the nearest hundredth, the roots are: \( x = 0.36, x = 1.21, x = 3.51, \) and \( x = 4.35 \)
The period is \( \pi \), so the general solution is approximately:
\( x = 0.36 + \pi k, k \in \mathbb{Z} \) or \( x = 1.21 + \pi k, k \in \mathbb{Z} \)

(c) \( 3 \cos x^2 = 2 \cos x + 1 \)
Graph \( y = 3 \cos x^2 - 2 \cos x - 1 \).
To the nearest hundredth, the roots are: \( x = 0, x = 1.91, \) and \( x = 4.37 \)
The period is \( 2\pi \), so the general solution is approximately:
\( x = 2\pi k, k \in \mathbb{Z} \) or \( x = 1.91 + 2\pi k, k \in \mathbb{Z} \) or
\( x = 4.37 + 2\pi k, k \in \mathbb{Z} \)
10. Solve each equation for \(0 \leq x < 2\pi\), then write the general solution.

a) \( \cos 3x = \frac{1}{2} \)

Graph \( y = \cos 3x - \frac{1}{2} \).

To the nearest hundredth, the roots are: \( x = 0.35, x = 1.75, \)
\( x = 2.44, x = 3.84, x = 4.54, \)
\( x = 5.93 \)
The period is \( \frac{2\pi}{3} \), so the general solution is approximately:
\[
x = 0.35 + \frac{2\pi}{3}k, \quad k \in \mathbb{Z}
\]
\[
x = 1.75 + \frac{2\pi}{3}k, \quad k \in \mathbb{Z}
\]

b) \( 1 - 4 \tan 3x = -7 \)

Graph \( y = 8 - 4 \tan 3x \).

To the nearest hundredth, the roots are: \( x = 0.37, x = 1.42, \)
\( x = 2.46, x = 3.51, x = 4.56, \)
\( x = 5.61 \)
The period is \( \frac{\pi}{3} \), so the general solution is approximately:
\[
x = 0.37 + \frac{\pi}{3}k, \quad k \in \mathbb{Z}
\]

11. The first two positive roots of the equation \( \sin 5x = \frac{1}{3} \) are \( x \approx 0.07 \) and \( x \approx 0.56 \). Determine the general solution of this equation. Explain how this solution is determined.

The period of the function is: \( \frac{2\pi}{5} \approx 1.26 \)

The graph of \( y = \sin 5x - \frac{1}{3} \) indicates that the two given roots are the only zeros of the function in the domain \( 0 \leq x \leq \frac{2\pi}{5} \).

So, the general solution is approximately: \( x = 0.07 + \frac{2\pi}{5}k, \quad k \in \mathbb{Z} \) or
\[
x = 0.56 + \frac{2\pi}{5}k, \quad k \in \mathbb{Z}
\]

12. Solve each equation over the given domain, then write the general solution.

a) \( \cos \pi x = 0 \) for \(-3 \leq x \leq 3 \)

Graph \( y = \cos \pi x \).

The graph is symmetrical about the \( y \)-axis.

The roots are: \( x = \pm 2.5, x = \pm 1.5, x = \pm 0.5 \)

The general solution is: \( x = 0.5 + k, \quad k \in \mathbb{Z} \)

b) \(-1 = 2 \sin 3\pi x \) for \(-1 \leq x \leq 1 \)

Graph \( y = 2 \sin 3\pi x + 1 \).

To the nearest hundredth, the roots are: \( x = -0.94, x = -0.72, \)
\( x = -0.28, x = -0.06, x = 0.39, x = 0.61 \)

The period is \( \frac{2\pi}{3} = \frac{2}{3} \), so the general solution is approximately:
\[
x = -0.72 + \frac{2k}{3}, \quad k \in \mathbb{Z} \) or \( x = -0.94 + \frac{2k}{3}, \quad k \in \mathbb{Z}
\]
13. Solve each equation over the set of real numbers.
   a) \(3\cos x = x^2 + 1\)
   
   Graph \(y = 3\cos x - x^2 - 1\).
   The solution is: \(x \approx \pm 0.91\).
   
   b) \(x^2 + 2 = 3x + \sin x\)
   
   Graph \(y = x^2 + 2 - 3x - \sin x\).
   The solution is: \(x \approx 0.60\) and \(x \approx 2.44\).
   
   c) \(\tan x + 2x^2 = 3x\)
   
   Graph \(y = \tan x + 2x^2 - 3x\).
   The solution is: \(x = 0, x \approx 0.84, x \approx 2.02,\) and \(x \approx 4.75\).

14. a) Solve \(\frac{\cos x}{1 - \sin x} = \frac{1 + \sin x}{\cos x}\) over the set of real numbers by
    graphing the two functions \(y = \frac{\cos x}{1 - \sin x}\) and \(y = \frac{1 + \sin x}{\cos x}\).
    
    What do you notice about the solution?
    
    The graphs coincide so the solution is all real values of \(x\), except for
    those values for which the denominators are 0.

    b) The equation in part a is called an *identity*. Why is that an
    appropriate name?
    
    One definition of identity is “exact likeness.” This is appropriate
    because one side of the equation is exactly the same as the other side.

15. Solve each equation over the set of real numbers.
   a) \(\sec x = \sqrt{4 - x^2}\)
   
   Graph \(y = \frac{1}{\cos x} - \sqrt{4 - x^2}\).
   The solution is: \(x \approx \pm 0.96\).
   
   b) \(\sin x + 2 = 2x\)
   
   Graph \(y = \sin x + 2 - 2x\).
   The solution is: \(x \approx 1.50\).
   
   c) \(\sin x = x^3\)
   
   Graph \(y = \sin x - x^3\).
   The solution is: \(x \approx \pm 0.93, x = 0\).

16. a) Solve each equation, and explain the results.
    i) \(\frac{\sin x}{x} = 1\)
    
    Graph \(y = \frac{\sin x}{x} - 1\).
    There is no real solution.
    When \(x = 0\), the left side is undefined
    
    ii) \(\sin x = x\)
    
    Graph \(y = \sin x - x\).
    The solution is: \(x = 0\).

    b) Why are the solutions in part a different?
    
    The solutions are different because in part i, \(x = 0\) is non-permissible;
    while in part ii, \(x = 0\) is permissible.
17. a) Solve each equation.
   i) \( \frac{\cos x}{x} = 1 \)
   ii) \( \cos x = x \)

   Graph \( y = \frac{\cos x}{x} - 1 \).
   The solution is: \( x \approx 0.74 \)

   Graph \( y = \cos x - x \).
   The solution is: \( x \approx 0.74 \)

b) Why are the solutions in part a the same?

   The solutions are the same because the equations are equivalent for \( x \neq 0 \).