7.1
1. Complete the table of values, then graph \( y = \left( \frac{1}{4} \right)^x \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y  )</td>
<td>16</td>
<td>4</td>
<td>1</td>
<td>0.25</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

The function is increasing.

b) Determine:

i) whether the function is increasing or decreasing

The function is increasing.

ii) the intercepts

There is no \( x \)-intercept; the \( y \)-intercept is 1.

iii) the equation of the asymptote

The asymptote has equation \( y = 0 \).

iv) the domain of the function

The domain is \( x \in \mathbb{R} \).

v) the range of the function

The range is \( y > 0 \).

7.2
2. a) Graph \( y = 3.5^x \) for \(-2 \leq x \leq 2\).

Make a table of values.
Write the coordinates to the nearest hundredth.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y  )</td>
<td>0.08</td>
<td>0.29</td>
<td>1</td>
<td>3.5</td>
<td>12.25</td>
</tr>
</tbody>
</table>

b) Determine:

i) whether the function is increasing or decreasing

The function is increasing.

ii) the intercepts

There is no \( x \)-intercept; the \( y \)-intercept is 1.

iii) the equation of the asymptote

The asymptote has equation \( y = 0 \).

iv) the domain of the function

The domain is \( x \in \mathbb{R} \).

v) the range of the function

The range is \( y > 0 \).
3. Use technology to graph each function below. For each graph:
   i) identify the intercepts
   ii) identify the equation of the asymptote and state why it is significant

a) \( y = 0.8^x \)

   i) There is no \( x \)-intercept.
   The \( y \)-intercept is 1.

   ii) The equation of the asymptote is \( y = 0 \).
   This is the line that the graph approaches as \( x \) increases.

b) \( y = 2.75^x \)

   i) There is no \( x \)-intercept.
   The \( y \)-intercept is 1.

   ii) The equation of the asymptote is \( y = 0 \).
   This is the line that the graph approaches as \( x \) decreases.

4. a) Sketch the graph of \( y = -\frac{1}{2}(3^x) - 1 \).

   Write the function as: \( y + 1 = -\frac{1}{2}(3^x) \)
   Compare \( y + 1 = -\frac{1}{2}(3^x) \) with \( y - k = c3^{dx-h} \):
   \( k = -1, c = -\frac{1}{2}, d = 2, \) and \( h = 0 \)
   Use the general transformation:
   \((x, y) \) corresponds to \( \left( \frac{x}{d} + h, cy + k \right) \)
   The point \((x, y) \) on \( y = 3^x \) corresponds to the point \( \left( \frac{x}{2}, -\frac{1}{2}y - 1 \right) \) on \( y + 1 = -\frac{1}{2}(3^x) \).
   Choose points \((x, y) \) on \( y = 3^x \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(\left( \frac{x}{2}, -\frac{1}{2}y - 1 \right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-2, \frac{1}{9}))</td>
<td>((-1, -\frac{19}{18}))</td>
</tr>
<tr>
<td>((-1, \frac{1}{3}))</td>
<td>((-\frac{1}{2}, -\frac{7}{6}))</td>
</tr>
<tr>
<td>((0, 1))</td>
<td>((0, -\frac{3}{2}))</td>
</tr>
<tr>
<td>((1, 3))</td>
<td>(\left( \frac{1}{2}, -\frac{5}{2} \right))</td>
</tr>
<tr>
<td>((2, 9))</td>
<td>(\left( 1, -\frac{11}{2} \right))</td>
</tr>
</tbody>
</table>
b) From the graph, identify:
   i) whether the function is increasing or decreasing
   The function is decreasing.

   ii) the intercepts
   There is no x-intercept. From the table, the y-intercept is −1.5.

   iii) the equation of the asymptote
   The asymptote has equation \( y = -1 \).

   iv) the domain of the function
   The domain of the function is \( x \in \mathbb{R} \).

   v) the range of the function
   The range of the function is \( y < -1 \).

7.3

5. Solve each equation.

   a) \( 4^x = 128 \)
   \[
   2^x = 2^7 \\
   2x = 7 \\
   x = 3.5
   \]

   b) \( 27^{x+1} = 81^{x-2} \)
   \[
   3^{3(x+1)} = 3^{4(x-2)} \\
   3x + 3 = 4x - 8 \\
   x = 11
   \]

   c) \( 8^{-3x} = \frac{2^x}{4} \)
   \[
   2^{-3x} = 2^{-2} \\
   -9x = x - 2 \\
   10x = 2 \\
   x = 0.2
   \]

   d) \( 9^x = 27 \sqrt[3]{3} \)
   \[
   3^{2x} = (3^3)(3^{\frac{1}{3}}) \\
   2x = 3.25 \\
   x = 1.625
   \]

   e) \( \frac{\sqrt{2}}{8} = 4^x \)
   \[
   (2^{\frac{1}{2}})(2^{-3}) = 2^{2x} \\
   -\frac{8}{3} = 2x \\
   x = -\frac{4}{3}
   \]

   f) \( \frac{1}{27^{x^2}} = 9^{x-2} \)
   \[
   3^{-2(x^2+1)} = 3^{2(x-2)} \\
   -3(x + 2) = 2(x - 2) \\
   -3x - 6 = 2x - 4 \\
   5x = -2 \\
   x = -0.4
   \]

6. Solve the equation \( 1.04^{2x} = 2 \). Give the solution to the nearest tenth.

   Use technology to graph \( y = 1.04^{2x} \) and \( y = 2 \).
   Determine the approximate x-coordinate of the point of intersection: 8.836
   The solution is: \( x \approx 8.8 \)
7. A new combine, used for harvesting wheat, costs $370,000. Its value depreciates by 10% each year. The value of the combine, \( v \) thousands of dollars, after \( t \) years can be modelled by this function:

\[ v = 370(0.9)^t \]

a) What is the value of the combine when it is 5 years old?

Give the answer to the nearest thousand dollars.

Use technology to graph \( y = 370(0.9)^x \) for \( 0 < x < 15 \).

When \( x = 5 \), \( y = 218.481 \)

After 5 years, the value of the combine is approximately $218,000.

b) When will the combine be worth $100,000?

Give the answer to the nearest half year.

Graph \( y = 100 \) on the same screen as \( y = 370(0.9)^x \).

When \( y = 100 \), \( x = 12.418 \)

The combine will be worth $100,000 after approximately 12.5 years.

8. A principal of $2500 is invested at 3% annual interest, compounded semi-annually. To the nearest year, how long will it be until the amount is $3000?

Use the formula:

\[ A = A_0 \left(1 + \frac{i}{n}\right)^{nt} \]

Substitute: \( A = 3000 \), \( A_0 = 2500 \), \( i = 0.03 \), \( n = 2 \)

\[ 3000 = 2500 \left(1.015\right)^{2t} \]

Use technology to graph \( y = 2500 \left(1.015\right)^{2x} \) and \( y = 3000 \) for \( 0 < x < 10 \).

When \( y = 3000 \), \( x = 6.122 \)

After approximately 6 years, the amount will be $3000.

7.4

9. a) Write each logarithmic expression as an exponential expression.

i) \( \log_3729 = 6 \)  

The base is 3.  
The exponent is 6.  
So, \( 729 = 3^6 \)

ii) \( \log_42\sqrt{2} = \frac{3}{4} \)  

The base is 4.  
The exponent is \( \frac{3}{4} \).  
So, \( 2\sqrt{2} = 4^{\frac{3}{4}} \)

iii) \( \log_5\frac{1}{\sqrt{25}} = -\frac{2}{3} \)  

The base is 5.  
The exponent is \( -\frac{2}{3} \).  
So, \( \frac{1}{\sqrt{25}} = 5^{-\frac{2}{3}} \)
b) Write each exponential expression as a logarithmic expression.

i) \(4^5 = 1024\)

The base is 4.
The logarithm is 5.
So, \(5 = \log_4{1024}\)

ii) \(5^{-4} = \frac{1}{625}\)

The base is 5.
The logarithm is \(-4\).
So, \(-4 = \log_5{\left(\frac{1}{625}\right)}\)

iii) \(6^{-\frac{1}{2}} = \frac{1}{\sqrt{6}}\)

The base is 6.
The logarithm is \(-\frac{1}{3}\).
So, \(-\frac{1}{3} = \log_6{\left(\frac{1}{\sqrt{6}}\right)}\)

RM 10. For each logarithm below, determine its exact value or use benchmarks to determine its approximate value to the nearest tenth.

a) \(\log_7{343}\)

\[= \log_7{(7^3)}\]
\[= 3\]

b) \(\log_8{100}\)

Identify powers of 8 close to 100.
\(8^2 = 64\) and \(8^3 = 512\)
So, \(2 < \log_8{100} < 3\)
An estimate is: \(\log_8{100} \approx 2.2\)
Check.
\(8^{2.2} \approx 97.00586026\)
\(8^{2.3} \approx 119.4282229\)
So, \(\log_8{100} \approx 2.2\)

c) \(\log_2{20}\)

Identify powers of 2 close to 20.
\(2^4 = 16\) and \(2^5 = 32\)
So, \(4 < \log_2{20} < 5\)
An estimate is: \(\log_2{20} \approx 4.3\)
Check.
\(2^{4.3} \approx 19.69831061\)
\(2^{4.4} \approx 21.11212657\)
So, \(\log_2{20} \approx 4.3\)

d) \(\log_4{\left(\frac{1}{32}\right)}\)

\[= \log_4{(2^{-5})}\]
\[= \log_4{(4^{\frac{1}{2}})^{-5}}\]
\[= \log_4{\left(4^{\frac{-5}{2}}\right)}\]
\[= -\frac{5}{2}\]

RM, US, CR1 11. a) Graph \(y = \log_x{x}\).

Determine values for \(y = 6^x\), then interchange the coordinates for the table of values for \(y = \log_x{x}\).

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y = \log_x{x})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{1}{6})</td>
<td>(-1)</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>
b) Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

There is no y-intercept. The x-intercept is 1.
The asymptote has equation \( x = 0 \).
The domain is \( x > 0 \). The range is \( y \in \mathbb{R} \).

c) How could you use the graph of \( y = \log_6 x \) to graph \( y = 6^x \)?
Use your strategy to graph \( y = 6^x \) on the grid in part a.

I reflect points on the graph of \( y = \log_6 x \) in the line \( y = x \), then join the points for the graph of \( y = 6^x \).

7.5
12. Write each expression as a single logarithm.

a) \( 3 \log x + \frac{1}{2} \log y - 2 \log z \)
\[ = \log x^3 + \log y^{\frac{1}{2}} - \log z^2 \]
\[ = \log \left( \frac{x^3y^{\frac{1}{2}}}{z^2} \right) \]

b) \( 4 + \log_2 3 \)
\[ = \log_{2^{4}} + \log_2 3 \]
\[ = \log_2 48 \]

b) \( 3 \log_2 x - 2 \log_2 y + 3 \)
\[ = \log_2 x^3 - \log_2 y^2 + \log_2 8 \]
\[ = \log_2 \left( \frac{8x^3}{y^2} \right) \]

13. Evaluate each expression.

a) \( 2 \log_6 6 - \log_6 18 + \log_6 8 \)
\[ = \log_6 6^2 + \log_6 8 - \log_6 18 \]
\[ = \log_6 \left( \frac{36 \cdot 8}{18} \right) \]
\[ = \log_6 16 \]
\[ = \log_6 4^2 \]
\[ = 2 \]

b) \( 2 \log_2 10 - \log_2 250 + \log_2 40 \)
\[ = \log_2 10^2 + \log_2 40 - \log_2 250 \]
\[ = \log_2 \left( \frac{100 \cdot 40}{250} \right) \]
\[ = \log_2 16 \]
\[ = \log_2 2^4 \]
\[ = 4 \]

7.6
14. Approximate the value of each logarithm, to the nearest thousandth.

a) \( \log_5 600 \)
\[ \log_5 600 = \frac{\log 600}{\log 5} \]
\[ = 3.9746 \ldots \]
\[ \approx 3.975 \]

b) \( \log_3 0.1 \)
\[ \log_3 0.1 = \frac{\log 0.1}{\log 3} \]
\[ = -2.0959 \ldots \]
\[ \approx -2.096 \]

c) \( \log_4 1.75 \)
\[ \log_4 1.75 = \frac{\log 1.75}{\log 4} \]
\[ = 0.4036 \ldots \]
\[ \approx 0.404 \]
15. Use technology to graph \( y = \log_9 x \). Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

Graph: \( y = \frac{\log x}{\log 9} \)

The \( x \)-intercept is 1. There is no \( y \)-intercept.
The equation of the asymptote is \( x = 0 \).
The domain of the function is \( x > 0 \). The range of the function is \( y \in \mathbb{R} \).

16. a) Sketch the graph of \( y = \log_5(3x - 6) + 3 \).

Write \( y = \log_5(3x - 6) + 3 \) as \( y - 3 = \log_5(3x - 2) \).
Compare \( y - 3 = \log_5(3x - 2) \) with \( y - k = c \log_d(x - h) \):
\( k = 3, c = 1, d = 3, \) and \( h = 2 \)
Use the general transformation: \((x, y)\) corresponds to \( \left( \frac{x}{d} + h, cy + k \right) \)

The point \((x, y)\) on \( y = \log_5 x \) corresponds to the point \( \left( \frac{x}{3} + 2, y + 3 \right) \)
on \( y = \log_5(3x - 6) + 3 \).

<table>
<thead>
<tr>
<th>((x, y))</th>
<th>(\left( \frac{x}{3} + 2, y + 3 \right))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\left( \frac{1}{25}, -2 \right))</td>
<td>(\left( \frac{151}{75}, 1 \right))</td>
</tr>
<tr>
<td>(\left( \frac{1}{5}, -1 \right))</td>
<td>(\left( \frac{31}{15}, 2 \right))</td>
</tr>
<tr>
<td>((1, 0))</td>
<td>(\left( \frac{7}{3}, 3 \right))</td>
</tr>
<tr>
<td>((5, 1))</td>
<td>(\left( \frac{11}{3}, 4 \right))</td>
</tr>
</tbody>
</table>
b) Identify the intercepts and the equation of the asymptote of the graph of \( y = \log_5(3x - 6) + 3 \), and the domain and range of this function.

There is no \( y \)-intercept.

For the \( x \)-intercept, substitute \( y = 0 \) in \( y = \log_5(3x - 6) + 3 \), then solve for \( x \).

\[
0 = \log_5(3x - 6) + 3
\]

\[
\log_5(3x - 6) = -3
\]

\[
x = 6 + \frac{1}{125}
\]

\[
x = 7.51
\]

There is no \( y \)-intercept.

The equation of the asymptote is \( x = 2 \).

The domain of the function is \( x > 2 \).

The range of the function is \( y \in \mathbb{R} \).

### 7.7

17. Solve, then verify each logarithmic equation.

a) \( 3 = \log_3(x + 5) + \log_3(x + 7) \)

\[
x > -5 \text{ and } x > -7; \ so \ x > -5
\]

\[
3 = \log_3((x + 5)(x + 7))
\]

\[
x^2 + 12x + 27 = 0
\]

\[
(x + 9)(x + 3) = 0
\]

\[
x = -9 \text{ or } x = -3
\]

\[
x = -9 \text{ is extraneous.}
\]

Verify: \( x = -3 \)

R.S. = \( \log_32 + \log_34 \)

\[
= 1 + 2
\]

\[
= 3
\]

\[
= \text{L.S.}
\]

The solution is verified.

b) \( \log x + \log (x + 1) = \log (7x - 8) \)

\[
x > 0, x > -1, x > \frac{8}{7}; \ so \ x > \frac{8}{7}
\]

\[
\log x(x + 1) = \log (7x - 8)
\]

\[
x(x + 1) = 7x - 8
\]

\[
x^2 - 6x + 8 = 0
\]

\[
(x - 2)(x - 4) = 0
\]

\[
x = 2 \text{ or } x = 4
\]

Verify: \( x = 2 \)

L.S. = \( \log 6 \)

R.S. = \( \log 6 \)

The solution is verified.

Verify: \( x = 4 \)

L.S. = \( \log 20 \)

R.S. = \( \log 20 \)

The solution is verified.

18. Solve each equation algebraically. Give the solution to the nearest hundredth.

a) \( 5^3x = 60 \)

\[
x = \frac{12}{5}
\]

\[
\log_5x = \log_512
\]

\[
\log 12
\]

\[
x = \frac{2.26}{\log 5}
\]

\[
x \approx 2.26
\]

b) \( 3^{x+4} = 5^{x+1} \)

\[
\log 3^{x+4} = \log 5^{x+1}
\]

\[
(x + 4)\log 3 = (x + 1)\log 5
\]

\[
x \log 3 + 4 \log 3 = x \log 5 + \log 5
\]

\[
x(\log 3 - \log 5) = \log 5 - 4 \log 3
\]

\[
x = \frac{\log 5 - 4 \log 3}{\log 3 - \log 5}
\]

\[
x \approx 5.45
\]
7.8

19. The pH of a solution can be described by the equation
\[ \text{pH} = -\log [H^+] \], where \([H^+]\) is the hydrogen-ion concentration in moles/litre. Determine the hydrogen-ion concentration in pure water with a pH of 7.

*Substitute pH = 7 in the equation:*
\[ \text{pH} = -\log [H^+] \]
\[ 7 = -\log [H^+] \quad \text{Write in exponential form.} \]
\[ [H^+] = 10^{-7} \]
The hydrogen-ion concentration of pure water is \(10^{-7}\) moles/litre.

7.9

20. Solve the equation: \(\ln (x - 3) + \ln 3 = \ln x\)
\[
\ln (x - 3) + \ln 3 = \ln x \quad x > 3 \\
\ln 3 = \ln x - \ln (x - 3) \\
\ln \left( \frac{x}{x - 3} \right) = \ln 3 \\
\frac{x}{x - 3} = 3 \\
x = 3x - 9 \\
2x = 9 \\
x = 4.5
\]
Verify the solution.