1. Multiple Choice Which logarithm has the greatest value?
   A. $\log_3 30$  B. $\log_2 9$  C. $\log_4 60$  D. $\log_7 360$

2. Multiple Choice Which function describes this graph?
   A. $f(x) = 2^{x+1} + 3$
   B. $f(x) = 2^{x-1} - 3$
   C. $f(x) = 2^{x+1} - 3$
   D. $f(x) = 2^{x-1} + 3$

3. a) Graph $y = 3^x$. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

Make a table of values, plot the points, then join them with a smooth curve.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>$-1$</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$1$</td>
<td>$3$</td>
</tr>
<tr>
<td>$2$</td>
<td>$9$</td>
</tr>
</tbody>
</table>

There is no $x$-intercept; the $y$-intercept is $1$; the equation of the asymptote is $y = 0$; the domain is $x \in \mathbb{R}$; and the range is $y > 0$. 
4. Solve each equation. Where necessary, give the solution to the nearest hundredth.

a) \(5^x = 400\)

\[
\log_5 5^x = \log_5 400 \\
x = \frac{\log 400}{\log 5} \\
x = 3.72
\]

b) \(8^{x+1} = 16^{x-2}\)

\[
2^{3(x+1)} = 2^{4(x-2)} \\
3x + 3 = 4x - 8 \\
x = 11
\]

c) \(\log_4(x - 8) + \log_4(x + 4) = 3\)

\[
x > 8 \text{ and } x > -4; \text{ so } x > 8 \\
\log_4(x - 8)(x + 4) = 3 \\
(x - 8)(x + 4) = 4^3 \\
x^2 - 4x - 96 = 0 \\
(x - 12)(x + 8) = 0 \\
x = 12 \text{ or } x = -8 \\
x = -8 \text{ is extraneous.} \\
\text{Verify } x = 12: \text{ L.S. = } \log_4 4 + \log_4 16 \text{ R.S. = R.S.} \\
= 1 + 2 \\
= 3 \\
= \text{R.S.} \\
\text{The solution is verified.}
\]

d) \(\log_5(3x + 4) = \log_5(x + 4) + \log_5(x - 2)\)

\[
x > -\frac{4}{3}, \text{ and } x > -4; \text{ so } x > 2 \\
\log_5(3x + 4) = \log_5(x + 4)(x - 2) \\
3x + 4 = (x + 4)(x - 2) \\
x^2 - x - 12 = 0 \\
(x - 4)(x + 3) = 0 \\
x = 4 \text{ or } x = -3 \\
x = -3 \text{ is extraneous.} \\
\text{Verify } x = 4: \text{ L.S. = } \log_5 16 \text{ R.S. = } \log_8 8 + \log_2 2 \text{ R.S. = } \log_1 16 \\
= \log_5 16 \\
\text{The solution is verified.} \]
5. a) Graph \( y = 5^x \):

Make a table of values, plot the points, then join them with a smooth curve.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1/25</td>
</tr>
<tr>
<td>-1</td>
<td>1/5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

b) Which transformations would be applied to the graph of \( y = 5^x \) so that the equation of its image is \( y = \frac{1}{2}(5^{x+4}) + 3 \)?

Write the function as: \( y - 3 = \frac{1}{2}(5^{-(x-4)}) \)

The transformations are: a vertical compression by a factor of \( \frac{1}{2} \); a reflection in the \( y \)-axis; a horizontal translation of 4 units right; and a vertical translation of 3 units up.

c) Graph \( y = \frac{1}{2}(5^{x+4}) + 3 \) on the grid in part a. Identify the intercepts and the equation of the asymptote of the graph, and the domain and range of the function.

Compare: \( y - 3 = \frac{1}{2}(5^{-(x-4)}) \) with \( y - k = c5^{ax-h} \):

\[ k = 3, \ c = \frac{1}{2}, \ d = -1, \ h = 4 \]

Use the general transformation: \((x, y)\) corresponds to \((x/\frac{d}{a} + h, cy + k)\)

The point \((x, y)\) on \( y = 5^x \) corresponds to the point \((-x + 4, \frac{1}{2}y + 3)\) on \( y - 3 = \frac{1}{2}(5^{-(x-4)}) \). Use the points from part a.

<table>
<thead>
<tr>
<th>( x )</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.02</td>
<td>3.1</td>
<td>3.5</td>
<td>5.5</td>
</tr>
</tbody>
</table>

There is no \( x \)-intercept.

For the \( y \)-intercept, substitute \( x = 0 \) in \( y = \frac{1}{2}(5^{x+4}) + 3 \)

\[ y = \frac{1}{2}(5^4) + 3 \]

\[ y = 315.5 \]

The equation of the asymptote is \( y = 3 \).

The domain is \( x \in \mathbb{R} \) and the range is \( y > 3 \).
6. How many monthly investments of $150 would have to be paid into a savings account that pays 3% annual interest, compounded monthly, to obtain an amount of $5000?

The amount of $5000 is the future value, so use:

\[ FV = \frac{R[(1 + i)^n - 1]}{i} \]

Substitute: \( FV = 5000; R = 150; i = \frac{0.03}{12} \), or 0.0025

\[
5000 = \frac{150(1 + 0.0025)^n - 1}{0.0025}
\]

\[
\frac{0.25}{3} = 1.0025^n - 1
\]

\[
1 + \frac{0.25}{3} = 1.0025^n
\]

\[
\log \left( \frac{3.25}{3} \right) = \log 1.0025^n
\]

\[
\log \left( \frac{3.25}{3} \right) = n \log 1.0025
\]

\[
n = \frac{\log \left( \frac{3.25}{3} \right)}{\log 1.0025}
\]

\[
n = 32.0570. . .
\]

The number of investments is approximately 32.

7. The population of India was about 1.3 billion people in 2017. The growth rate is about 1.13% per year. Suppose this trend continues. To the nearest tenth of a billion, what will the population be in 2067?

To determine the value of \( k \), use the formula: \( kt = \ln \left( \frac{A}{A_0} \right) \)

Substitute: \( \frac{A}{A_0} = 101.13\% \), or 1.0113, and \( t = 1 \)

\[
k(1) = \ln (1.0113)
\]

\[
k = \ln (1.0113)
\]

To determine the population in 2067, use the formula: \( A = A_0e^{kt} \)

Substitute: \( A_0 = 1.3, k = \ln (1.0113), t = 2067 - 2017 \), or 50

\[
A = 1.3e^{50 \ln (1.0113)}
\]

\[
A = 2.2800. . .
\]

In 2067, the population will be approximately 2.3 billion.