Lesson 7.7 Exercises, pages 466–471

Exercises

3. Write each expression as a single logarithm.

a) \( \log 24 - \log 3 \)

Use the quotient law.
\[ \log 24 - \log 3 = \log \left( \frac{24}{3} \right) = \log 8 \]

b) \( \log 2 + \log 4 \)

Use the product law.
\[ \log 2 + \log 4 = \log (2 \cdot 4) = \log 8 \]

c) \( 2 \log 4 - \log 4 \)

Subtract.
\[ 2 \log 4 - \log 4 = \log 4 \]

d) \( 3 \log 2 + 2 \log 3 \)

Use the power law then the product law.
\[ 3 \log 2 + 2 \log 3 = \log 2^3 + \log 3^2 \]
\[ = \log (8 \cdot 9) = \log 72 \]

4. Write each expression as a single logarithm.

a) \( \log_b(x^2 + 2x) \)

Use the product law.
\[ \log_b(x + 2) + \log_b x \]
\[ = \log_b(x + 2)x \]
\[ = \log_b(x^2 + 2x) \]

c) \( \log_b(x - 5) + \log_b(x + 3) \)

Use the product law.
\[ \log_b(x - 5) + \log_b(x + 3) \]
\[ = \log_b((x - 5)(x + 3)) \]
\[ = \log_b(x^2 - 2x - 15) \]

d) \( \log_b(x - 7) + \log_b(x + 7) \)

Use the product law.
\[ \log_b(x - 7) + \log_b(x + 7) \]
\[ = \log_b((x - 7)(x + 7)) \]
\[ = \log_b(x^2 - 49) \]

e) \( \log_b(3x - 2) - 2 \log_b x \)

Use the power law then the quotient law.
\[ \log_b(3x - 2) - 2 \log_b x \]
\[ = \log_b(\frac{(3x - 2)}{x^2}) \]

e) \( \log_b(3x - 2) - 2 \log_b x \)

Use the power law then the quotient law.
\[ \log_b(3x - 2) - 2 \log_b x \]
\[ = \log_b(\frac{(3x - 2)}{x^2}) \]

5. Use substitution or logical reasoning to determine whether \( x = 2 \) is a root of each equation.

a) \( \log_5x + \log_5 2x = 3 \)

Use substitution.
\[ \text{L.S.} = \log_5 2 + \log_5 4 \]
\[ = 1 + 2 \]
\[ = 3 \]
\[ = \text{R.S.} \]
So, \( x = 2 \) is a root.

b) \( \log (x - 4) + \log (x - 7) = 1 \)

Use logical reasoning.
When \( x = 2 \) is substituted in \( \log (x - 4) \), it becomes \( \log (-2) \), which is not defined.
So, \( x = 2 \) is not a root.

Use logical reasoning.
When \( x = 2 \) is substituted in \( \log(x - 2) \), it becomes \( \log 0 \), which is not defined.
So, \( x = 2 \) is not a root.
6. Explain why \( \frac{\log 80}{\log 4} \neq \log 20 \). Use a calculator to verify.

I cannot divide the logarithms of 2 numbers by determining the logarithm of the quotient of the numbers.

L.S. = \( \frac{\log 80}{\log 4} \)
R.S. = \( \log 20 \)

= 3.1609 . . .
= 1.3010 . . .

Since the left side is not equal to the right side, \( \frac{\log 80}{\log 4} \neq \log 20 \)

7. Solve each exponential equation. Give the solution to the nearest hundredth.

a) \( 60 = 3^{x+1} \)
\[ \log 60 = \log 3^{x+1} \]
\[ \log 60 = x + 1 \]
\[ x = \log 60 - 1 \]
\[ x = \frac{\log 60}{\log 3} - 1 \]
\[ x \approx 2.73 \]

b) \( 5^{x-3} = 200 \)
\[ \log 5^{x-3} = \log 200 \]
\[ x - 3 = \log 200 \]
\[ x = \log 200 + 3 \]
\[ x = \frac{\log 200}{\log 5} + 3 \]
\[ x \approx 6.29 \]

c) \( 4^{2x-1} = \frac{1}{5} \)
\[ (2x - 1) \log 4 = \log \frac{1}{5} \]
\[ 2x - 1 = \frac{\log \frac{1}{5}}{\log 4} \]
\[ 2x = \frac{\log \frac{1}{5}}{\log 4} + 1 \]
\[ x = \frac{1}{2} \left( \frac{\log \frac{1}{5}}{\log 4} + 1 \right) \]
\[ x \approx -0.08 \]

8. Consider the exponential equation \( 3^x = 30 \).

a) Solve this equation algebraically. Give the root to the nearest hundredth.
\[ 3^x = 30 \]
\[ \log 3^x = \log 30 \]
\[ x = \frac{\log 30}{\log 3} \]
\[ x \approx 3.10 \]

b) Verify the solution by graphing.

In graphing technology, input: \( y = 3^x \) and \( y = 30 \)
Determine the approximate \( x \)-coordinate of the point of intersection of the graphs: 3.096
The solution is verified.
9. Solve, then verify each logarithmic equation.

   a) \( 4 = \log_2 x + \log_2 (x + 6) \)

   \[
   \begin{align*}
   x > 0 \text{ and } x > -6; \text{ so } x > 0 \\
   4 & = \log_2 x(x + 6) \\
   2^4 & = x(x + 6) \\
   x^2 + 6x - 16 & = 0 \\
   (x - 2)(x + 8) & = 0 \\
   x = 2 \text{ or } x = -8 \\
   x = -8 \text{ is extraneous.} \\
   \text{Verify } x = 2: \\
   \text{R.S.} & = \log_2 2 + \log_2 8 \\
   & = 1 + 3 \\
   & = 4 \\
   & = \text{L.S.} \\
   \text{The solution is verified.}
   \end{align*}
   \]

   b) \( \log_5 x + \log_5 (x - 16) = 2 \)

   \[
   \begin{align*}
   x > 0 \text{ and } x > 16; \text{ so } x > 16 \\
   \log_5 x(x - 16) & = 2 \\
   x(x - 16) & = 5^2 \\
   x^2 - 16x - 36 & = 0 \\
   (x - 18)(x + 2) & = 0 \\
   x = 18 \text{ or } x = -2 \\
   x = -2 \text{ is extraneous.} \\
   \text{Verify } x = 18: \\
   \text{L.S.} & = \log_5 18 + \log_5 2 \\
   & = \log_5 36 \\
   & = 2 \\
   & = \text{R.S.} \\
   \text{The solution is verified.}
   \end{align*}
   \]

10. Consider the equation \( 4^{x+2} = 8^{x-1} \).

   a) Solve this equation algebraically using logarithms.

   \[
   \begin{align*}
   \log_2 4^{x+2} & = \log_2 8^{x-1} \\
   \log_2 2^{2(x+2)} & = \log_2 2^{3(x-1)} \\
   2(x + 2) & = 3(x - 1) \\
   2x + 4 & = 3x - 3 \\
   x & = 7
   \end{align*}
   \]

   b) Solve this equation using a different algebraic strategy.

   \[
   \begin{align*}
   4^{x+2} & = 8^{x-1} \quad \text{Write each side with base 2.} \\
   2^{2(x+2)} & = 2^{3(x-1)} \quad \text{Equate exponents.} \\
   2x + 4 & = 3x - 3 \\
   x & = 7
   \end{align*}
   \]

   c) Which strategy in parts a and b is more efficient?

   Justify your answer.

   The strategy in part b is more efficient because there are fewer steps in the solution. This strategy only works because both bases can be changed to the same base to allow the exponents to be equated.
11. Solve, then verify each logarithmic equation.

a) \( \log_6 48 = \log_6 (x + 7) + \log_6 (x - 1) \)
   
   \( x > -7 \) and \( x > 1; \) so \( x > 1 \)
   
   \( \log_6 48 = \log_6(x + 7)(x - 1) \)
   
   \( 48 = (x + 7)(x - 1) \)
   
   \( 48 = x^2 + 6x - 7 \)
   
   \( x^2 + 6x - 55 = 0 \)
   
   \( (x - 5)(x + 11) = 0 \)
   
   \( x = 5 \) or \( x = -11 \)
   
   \( x = -11 \) is extraneous.
   
   Verify \( x = 5: \)
   
   \( \text{L.S.} = \log_6 48 \)
   
   \( = \log_6 48 \)
   
   \( = \text{R.S.} \)
   
   The solution is verified.

b) \( \log (2x + 4) - \log (x + 2) = \log (x + 1) \)

   \( x > -2, x > -2, \) and \( x > -1; \) so \( x > -1 \)

   \( \log \left( \frac{2x + 4}{x + 2} \right) = \log (x + 1) \) Factor the left side, then divide.

   \( \log 2 = \log (x + 1) \)
   
   \( 2 = x + 1 \)
   
   \( x = 1 \)
   
   Verify \( x = 1: \)

   \( \text{L.S.} = \log 6 - \log 3 \quad \text{R.S.} = \log 2 \)

   \( = \log 2 \)

   The solution is verified.

12. Solve each exponential equation algebraically. Write the solution to the nearest hundredth.

a) \( 200 = 5(2^{x - 1}) \)

   \( 40 = 2^{x - 1} \)
   
   \( \log_2 40 = \log_2 2^{x - 1} \)
   
   \( \log_2 40 = x - 1 \)
   
   \( x = \log_2 40 + 1 \)
   
   \( x = \frac{\log 40}{\log 2} + 1 \)
   
   \( x \approx 6.32 \)

b) \( 4^x = 6^{x - 2} \)

   \( \log 4^x = \log 6^{x - 2} \)
   
   \( x \log 4 = (x - 2) \log 6 \)
   
   \( x \log 4 = x \log 6 - 2 \log 6 \)
   
   \( x \log 4 - x \log 6 = -2 \log 6 \)
   
   \( x(\log 4 - \log 6) = -2 \log 6 \)
   
   \( x = \frac{-2 \log 6}{\log 4 - \log 6} \)
   
   \( x \approx 8.84 \)

c) \( 5^{x+2} = 10^{x-1} \)

   \( \log 5^{x+2} = \log 10^{x-1} \)
   
   \( (x + 2) \log 5 = x - 1 \)
   
   \( x \log 5 + 2 \log 5 = x - 1 \)
   
   \( x \log 5 - x = -2 \log 5 - 1 \)
   
   \( x(\log 5 - 1) = -2 \log 5 - 1 \)
   
   \( x = \frac{-2 \log 5 - 1}{\log 5 - 1} \)
   
   \( x \approx 7.97 \)

d) \( 3(2^x) = 6^{x-1} \)

   \( \log 3(2^x) = \log 6^{x-1} \)
   
   \( \log 3 + \log 2^x = (x - 1) \log 6 \)
   
   \( \log 3 + x \log 2 = x \log 6 - \log 6 \)
   
   \( x \log 2 - x \log 6 = -\log 6 - \log 3 \)
   
   \( x(\log 2 - \log 6) = -\log 6 - \log 3 \)
   
   \( x = \frac{-\log 6 - \log 3}{\log 2 - \log 6} \)
   
   \( x \approx 2.63 \)
13. Solve, then verify each logarithmic equation.

a) \( \log (2x - 7) + \log (x - 1) = \log (x + 1) + \log (x - 3) \)

\[ x > 3.5, x > 1, x > -1, \text{ and } x > 3; \text{ so } x > 3.5 \]
\[ \log (2x - 7)(x - 1) = \log (x + 1)(x - 3) \]
\[ (2x - 7)(x - 1) = (x + 1)(x - 3) \]
\[ 2x^2 - 9x + 7 = x^2 - 2x - 3 \]
\[ x^2 - 7x + 10 = 0 \]
\[ (x - 5)(x - 2) = 0 \]
\[ x = 5 \text{ or } x = 2 \]
\[ x = 2 \text{ is extraneous, because } x > 3.5 \]

Verify \( x = 5 \):
\[ \text{L.S.} = \log 3 + \log 4 \quad \text{R.S.} = \log 6 + \log 2 \]
\[ = \log 12 \quad = \log 12 \]

The solution is verified.

b) \( \log_2(x + 4) - \log_2(x - 2) = 1 + \log_2(2x - 1) - \log_2(x + 1) \)

\[ x > -4, x > 2, x > 0.5, \text{ and } x > -1; \text{ so } x > 2 \]
\[ \log_2(x + 4) - \log_2(x - 2) = \log_2 2 + \log_2(2x - 1) - \log_2(x + 1) \]
\[ \frac{\log_2(x + 4)}{\log_2(x - 2)} = \log_2 \left( \frac{2(2x - 1)}{x + 1} \right) \]
\[ x + 4 = \frac{2(2x - 1)}{x + 1} \]
\[ x - 2 = \frac{2x - 2}{x + 1} \]
\[ (x + 4)(x + 1) = (4x - 2)(x - 2) \]
\[ x^2 + 5x + 4 = 4x^2 - 10x + 4 \]
\[ 3x^2 - 15x = 0 \]
\[ 3x(x - 5) = 0 \]
\[ x = 0 \text{ or } x = 5 \]
\[ x = 0 \text{ is extraneous, because } x > 2 \]

Verify \( x = 5 \):
\[ \text{L.S.} = \log_2 9 - \log_2 3 \quad \text{R.S.} = \log_2 2 + \log_2 9 - \log_2 6 \]
\[ = \log_2 3 \quad = \log_2 18 - \log_2 6 \]
\[ = \log_2 3 \]

The solution is verified.

14. a) Solve the equation \( 10^x = k \), where \( k > 0 \).

\[ 10^x = k \]
\[ \log 10^x = \log k \]
\[ x \log 10 = \log k \quad \text{Substitute: } \log 10 = 1 \]
\[ x = \log k \]

b) Explain the result.

\( x \) is the power to which 10 is raised to get \( k \).
And, by definition, the power to which 10 is raised to get \( k \) is \( \log k \).
15. Consider the equation \( \log_2(x - 1) + \log_2(x - 3) = 1 \).

a) Use graphing technology to solve the equation.
Write the solution to the nearest hundredth.

Write each logarithm to base 10 by dividing by \( \log 2 \).
Input: \( y = \frac{\log (x - 1)}{\log 2} + \frac{\log (x - 3)}{\log 2} \), and \( y = 1 \)
The approximate \( x \)-coordinate of the point of intersection is: 3.732
The solution is: \( x \approx 3.73 \)

b) Solve the equation algebraically to determine the exact value of \( x \).

\( x > 1 \) and \( x > 3 \); so \( x > 3 \)
\( \log_2(x - 1) + \log_2(x - 3) = 1 \)
\( \log_2(x - 1)(x - 3) = \log_22 \)
\( (x - 1)(x - 3) = 2 \)
\( x^2 - 4x + 1 = 0 \)

Use the quadratic formula.
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]
Substitute: \( a = 1, \ b = -4, \ c = 1 \)
\[
x = \frac{4 \pm \sqrt{16 - 4}}{2}
\]
\[
x = \frac{4 \pm 2\sqrt{3}}{2}
\]
\[
x = 2 \pm \sqrt{3}
\]
Since \( x > 3 \), \( x = 2 - \sqrt{3} \) is extraneous.
The solution is: \( x = 2 + \sqrt{3} \)