Lesson 7.3 Exercises, pages 415–419

Exercises

A

3. Write each number as a power of 2.
   a) 16   b) 128   c) \(\frac{1}{32}\)   d) 1   e) \(\frac{1}{64}\)   f) \(\frac{1}{128}\)
   \[= 2^4 = 2^7 = 2^{-5} = 2^0 = 2^{-4} = 2^{-7}\]

4. Which numbers below can be written as powers of 5? Write each number you identify as a power of 5.
   a) 125   b) 10   c) \(\frac{1}{25}\)   d) 1   e) \(\frac{1}{1000}\)   f) \(\frac{1}{625}\)
   \[= 5^3\text{ cannot be written as a power of 5 } = 5^{-2} = 5^0\text{ cannot be written as a power of 5}\]

5. Write each number as a power of 3.
   a) \(\sqrt[3]{9}\)   b) \(\sqrt[4]{243}\)   c) \(\frac{\sqrt[3]{3}}{3}\)   d) \(27\sqrt[3]{3}\)   e) \(\frac{\sqrt[3]{27}}{3}\)   f) \(\frac{\sqrt[3]{9}}{3}\)
   \[= 9^{\frac{1}{3}} = 243^{\frac{1}{4}} = 3^{\frac{1}{3}} \cdot 3^{-1} = 3^{\frac{1}{3}} \cdot 3^1 = \frac{1}{3^{\frac{1}{3}}} = \frac{3^{\frac{1}{2}}}{3^{\frac{1}{3}}}\]

B

6. Solve each equation.
   a) \(2^x = 256\)   b) \(81 = 3^{x+1}\)   c) \(3^x = 9^{x-2}\)
   \[2^x = 2^8\]
   \[x = 8\]
   \[3^x = 3^{x+1}\]
   \[4 = x + 1\]
   \[x = 3\]
   \[3^x = (3^2)^{x-2}\]
   \[x = 2(x - 2)\]
   \[x = 2x - 4\]
   \[x = 4\]

   d) \(4^{x-1} = 2^{x+3}\)
   \(2(x - 1) = x + 3\)
   \(2x - 2 = x + 3\)
   \[x = 5\]

   e) \(8^{2x} = 16^{x+3}\)
   \((2^3)^x = (2^4)^{x+3}\)
   \(3(2x) = 4(x + 3)\)
   \(6x = 4x + 12\)
   \(2x = 12\)
   \[x = 6\]

   f) \(9^{x+1} = 243^{x+3}\)
   \((3^2)^{x+1} = (3^3)^{x+3}\)
   \(2(x + 1) = 5(x + 3)\)
   \(2x + 2 = 5x + 15\)
   \[-3x = 13\]
   \[x = \frac{-13}{3}\]

   g) \(8^{2x+3} = 16^{2x-3}\)
   \((2^3)^{2x+3} = (2^4)^{2x-3}\)
   \(3(2x + 3) = 4(2x - 3)\)
   \(6x + 9 = 8x - 12\)
   \(2x = 21\)
   \[x = \frac{21}{2}\]

   h) \(81^{x-2} = 243^{3x+1}\)
   \((3^4)^{x-2} = (3^5)^{3x+1}\)
   \(4(3 - x) = 5(3x + 1)\)
   \(12 - 4x = 15x + 5\)
   \[7 = 19x\]
   \[x = \frac{7}{19}\]
7. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) \(10 = 2^x\)

Graph: \(y = 2^x - 10\)
The approximate zero is 3.322
\(x \approx 3.3\)

b) \(3^x = 100\)

Graph: \(y = 100 - 3^x\)
The approximate zero is 4.192
\(x \approx 4.2\)

c) \(5^{3x-1} = 200\)

Graph: \(y = 5^{3x-1} - 200\)
The approximate zero is 1.431
\(x \approx 1.4\)

d) \(3^{x+1} = 50\)

Graph: \(y = 50 - 3^{x+1}\)
The approximate zero is 2.561
\(x \approx 2.6\)

e) \(30 = 2^{x-1}\)

Graph: \(y = 2^{x-1} - 30\)
The approximate zero is 5.907
\(x \approx 5.9\)

8. Explain why the equation \(4^x = -2\) does not have a real solution.

Verify, graphically, that there is no solution.

The value of a power with a positive base can never be negative, so the equation does not have a real solution. When I graph \(y = -2 - 4^x\), the graph does not have an \(x\)-intercept.

9. Solve each equation.

a) \(2^x = 8\sqrt{2}\)

\(2^x = 2^3 \cdot 2^{1/2}\)
\(2^x = 2^{3 + 1/2}\)
\(x = 3 + \frac{1}{2}\)
\(x = \frac{7}{2}\)

b) \(81\sqrt{3} = 3^x\)

\(3^x \cdot \sqrt{3} = 3^x\)
\(3^{x+\frac{1}{2}} = 3^x\)
\(4 + \frac{1}{2} = x\)
\(x = \frac{9}{2}\)

c) \(2^{x+1} = 2\sqrt{4}\)

\(2^{x+1} = 2 \cdot 2^{1/2}\)
\(2^{x+1} = 2(2^{1/2})\)
\(2^{x+1} = 2^{1.5}\)
\(2^{x+1} = 2^{1 + 1/2}\)
\(x + 1 = 1 + \frac{2}{3}\)
\(x = \frac{2}{3}\)

d) \(9^x = \sqrt{27}\)

\((3^2)^x = (3^{3/2})^x\)
\(2x = \frac{3}{2}\)
\(x = \frac{3}{4}\)

e) \(\sqrt{216} = 36^{x-1}\)

\(216^{1/2} = 6^{2(x-1)}\)
\((6^2)^{1/2} = 6^{2x-1}\)
\(\frac{3}{4} = 2x - 2\)
\(2x = \frac{11}{4}\)
\(x = \frac{11}{8}\)

f) \((\sqrt{7})^{x+1} = \sqrt{49}\)

\(7^{x+1/2} = (7^2)^{1/2}\)
\(\frac{1}{2}x + \frac{1}{2} = 2\)
\(3x + 3 = 4\)
\(3x = 1\)
\(x = \frac{1}{3}\)
10. Solve each equation.

a) \((\frac{1}{3})^x = 2^x\)

\((2^{-x}) = 2^x\)
\((-2)(3) = x\)
\(x = -6\)

b) \(5^x = \frac{\sqrt{25}}{25}\)

\(5^x = (5^3) \cdot 5^{-2}\)
\(5^x = \frac{5^3}{5^2}\)
\(x = \frac{2}{3} - 2\)
\(x = \frac{-4}{3}\)

c) \(\sqrt[3]{49} = 7^{x+1}\)

\((7^2)^{1/2} \cdot 7^{-3} = 7^{x+1}\)
\(\frac{7^2}{7^3} = 7^{x+1}\)
\(\frac{7}{3} - 3 = x + 1\)
\(-\frac{7}{3} = x + 1\)
\(x = \frac{10}{3}\)

d) \((\frac{1}{5})^x = 3\sqrt{27}\)

\((3^{-2})^y = 3' \cdot (3')^2\)
\(3^{-2x} = 3^{1+\frac{1}{2}}\)
\(-2x = 1 + \frac{3}{2}\)
\(-2x = \frac{5}{2}\)
\(x = \frac{-5}{4}\)

e) \(8^{1-x} = \frac{\sqrt[4]{16}}{4}\)

\(2^{3(1-x)} = (2^3)^{1/2} \cdot 2^{-2}\)
\(2^{3(x-1)} = 2^{\frac{3}{2}} \cdot 2^{-2}\)
\(3 - 3x = \frac{4}{3} - 2\)
\(-3x = \frac{-11}{3}\)
\(x = \frac{11}{9}\)

f) \((\frac{1}{5})^{x+1} = (\sqrt[16]{16})^x\)

\((2^{-3})^{x+1} = (2^{\frac{3}{4}})^x\)
\(-3x = \frac{4}{3}x\)
\(-\frac{13}{3}x = 3\)
\(x = \frac{-9}{13}\)

g) \(\frac{25^x}{\sqrt{125}} = \frac{5^3x}{\sqrt{25}}\)

\(\frac{5^2x}{5^2} = \frac{5^3x}{5^3}\)
\(2x - \frac{3}{2} = 3x - \frac{2x}{3}\)
\(12x - 9 = 18x - 4x\)
\(2x = -9\)
\(x = \frac{-9}{2}\)

h) \(\frac{\sqrt[4]{8^{x+1}}}{4^{2x}} = \frac{64^{x-2}}{\sqrt[16]{16}}\)

\(\frac{2^{\frac{3}{2}(x+1)}}{2^{2(2x) - 2}} = \frac{2^{6(x-2)}}{2^{4x}}\)
\(\frac{3(x+1)}{2} - 4x = 6(x - 2) - 4x\)
\(9x + 9 - 24x = 36x - 72 - 8x\)
\(43x = 81\)
\(x = \frac{81}{43}\)

11. Use graphing technology to solve each equation. Give the solution to the nearest tenth.

a) \(2 = 1.05^x\)

Graph: \(y = 1.05^x - 2\)
The approximate zero is 14.207
\(x \approx 14.2\)

d) \(3(2^x) = 64\)

Graph: \(y = 64 - 3(2^x)\)
The approximate zero is 4.415
\(x \approx 4.4\)

b) \(2^{-\frac{x}{3}} = 0.4\)

Graph: \(y = 0.4 - 2^{-\frac{x}{3}}\)
The approximate zero is 6.610
\(x \approx 6.6\)

e) \(2(3^{2x-1}) = 3(4^{x+2})\)

Graph: \(y = 2(3^{2x-1}) - 3(4^{x+2})\)
The approximate zero is 5.274
\(x \approx 5.3\)

f) \((2^{2x+3}) = (3^{2x}(5)^{x+4})\)

Graph: \(y = (2^{2x+3}) - (3^{2x}(5)^{x+4})\)
The approximate zero is -1.801
\(x \approx -1.8\)
12. A principal of $600 was invested in a term deposit that pays 5.5% annual interest, compounded semi-annually. To the nearest tenth of a year, when will the amount be $1000?

Use: \( A = A_0 \left(1 + \frac{i}{n}\right)^{nt} \)

Substitute: \( A = 1000, A_0 = 600, i = 0.055, n = 2 \)

\[
1000 = 600 \left(1 + \frac{0.055}{2}\right)^{2t}
\]

Graph \( y = 600 \left(1 + \frac{0.055}{2}\right)^{2t} - 1000 \), then determine the zero of the function.

The approximate zero is 9.415

It will take approximately 9.4 years for the term deposit to amount to $1000.

13. a) To the nearest year, how long will it take an investment of $500 to double at each annual interest rate, compounded annually?

i) 4%  
ii) 6%  
iii) 8%  
iv) 9%  
v) 12%

Use: \( A = A_0 \left(1 + \frac{i}{n}\right)^{nt} \)

Substitute: \( A = 1000, A_0 = 500, n = 1 \)

\[
1000 = 500 \left(1 + \frac{i}{1}\right)^{t}
\]

Use this expression below.

i) Substitute: \( i = 0.04 \)

\[
2 = (1 + 0.04)^t \quad 2 = (1 + 0.06)^t \quad 2 = 1.04^t \quad 2 = 1.06^t \quad \text{Graph: } y = 1.04^t - 2 \quad \text{Graph: } y = 1.06^t - 2
\]

The approximate zero is: 17.673

It will take approximately 18 years.

iii) Substitute: \( i = 0.08 \)

\[
2 = (1 + 0.08)^t \quad 2 = (1 + 0.09)^t \quad 2 = 1.08^t \quad 2 = 1.09^t \quad \text{Graph: } y = 1.08^t - 2 \quad \text{Graph: } y = 1.09^t - 2
\]

The approximate zero is: 9.006

It will take approximately 9 years.

b) What pattern is there in the interest rates and times in part a?

The product of each interest rate as a percent and time in years is 72.
14. When light passes through glass, the intensity is reduced by 5%.

a) Determine a function that models the percent of light, \( P \), that passes through \( n \) layers of glass.

For 0 layers of glass, the percent of light is: \( P = 100 \)
For 1 layer of glass, the percent of light is: \( P = 100(0.95) \)
For 2 layers of glass, the percent of light is: \( P = 100(0.95)^2 \)
For 3 layers of glass, the percent of light is: \( P = 100(0.95)^3 \)
For \( n \) layers of glass, the percent of light is: \( P = 100(0.95)^n \)

b) Determine how many layers of glass are needed for only 25% of light to pass through.

Solve the equation: \( 25 = 100(0.95)^n \)
Graph a related function: \( y = 100(0.95)^n - 25 \)
The approximate zero of the function is: 27.026815
So, 27 layers of glass are needed.

15. Solve each equation, then verify the solution graphically.

a) \( 2^{(x^2)} = 16 \)
\[
\begin{align*}
2^{(x^2)} &= 2^4 \\
x^2 &= 4 \\
x &= \pm 2
\end{align*}
\]
The graph of \( y = 16 - 2^{(x^2)} \) has \( x \)-intercepts 2 and -2.

b) \( 9^{x+4} = 3^{(x^2)} \)
\[
\begin{align*}
3^{2(x+4)} &= 3^{(x^2)} \\
2x + 8 &= x^2 \\
x^2 - 2x - 8 &= 0
\end{align*}
\]
\((x - 4)(x + 2) = 0\)
\(x = 4 \) or \( x = -2\)

A graph of \( y = 3^{(x)} - 9^{x+4} \) has \( x \)-intercepts -2 and 4.

16. For what values of \( k \) does the equation \( 9^{(x^3)} = 27^{x+k} \) have no real solution?

\[
\begin{align*}
9^{(x^3)} &= 27^{x+k} \\
3^{(2x^3)} &= 3^{(3x+k)} \\
2x^3 &= 3x + 3k \\
2x^3 - 3x - 3k &= 0
\end{align*}
\]
For no real roots, the discriminant is less than 0.
\(( -3 )^2 - 4(2)(-3k) < 0\)
\((-3)^2 < 4(2)(-3k)\)
\(9 < -24k\)
\(k < -\frac{9}{24}\) or \(-\frac{3}{8}\)

The equation has no real solution when \( k < -\frac{3}{8} \).