4.1

1. Here is the graph of \( y = f(x) \). Sketch the graph of each function below. Write the domain and range of each translation image.

   a) \( y - 2 = f(x) \)
   Each point on the graph of \( y = f(x) \) is translated 2 units up.
   From the graph, the domain is: \( x \in \mathbb{R} \); the range is: \( y \geq -1 \)

   b) \( y = f(x - 3) \)
   Each point on the graph of \( y = f(x) \) is translated 3 units right.
   From the graph, the domain is: \( x \in \mathbb{R} \); the range is: \( y \geq -3 \)

   c) \( y + 3 = f(x + 4) \)
   Each point on the graph of \( y = f(x) \) is translated 4 units left and 3 units down. From the graph, the domain is: \( x \in \mathbb{R} \); the range is: \( y \geq -3 \)

2. The graph of the function \( y = x^3 - 2 \) is translated 3 units left and 4 units up. Write the equation of the translation image.

   The equation of the translation image has the form \( y - k = (x - h)^3 - 2 \), with \( h = -3 \) and \( k = 4 \)
   So, the equation is: \( y - 4 = (x + 3)^3 - 2 \), which simplifies to \( y = (x + 3)^3 + 2 \)

4.2

3. On the graph of \( y = g(x) \), sketch the graph of each function below. Write the domain and range of each reflection image.

   a) \( y = -g(x) \)
   Reflect points on \( y = g(x) \) in the \( x \)-axis:
   \((-2, -10) \) becomes \((-2, 10) \)
   \((0, -2) \) becomes \((0, 2) \)
   \((2, 6) \) becomes \((-2, -6) \)
   Draw a smooth curve through the points for the graph of \( y = -g(x) \).
   The domain is: \( x \in \mathbb{R} \)
   The range is: \( y \in \mathbb{R} \)

   b) \( y = g(-x) \)
   Reflect points on \( y = g(x) \) in the \( y \)-axis:
   \((-2, -10) \) becomes \((2, -10) \)
   \((0, -2) \) stays as \((0, -2) \)
   \((2, 6) \) becomes \((-2, 6) \)
   Draw a smooth curve through the points for the graph of \( y = g(-x) \).
   The domain is: \( x \in \mathbb{R} \)
   The range is: \( y \in \mathbb{R} \)
4. The graph of \( f(x) = (x - 2)^3 - 4 \) was reflected in the \( x \)-axis and its image is shown. What is an equation of the image? Explain.

When the graph of \( y = f(x) \) is reflected in the \( x \)-axis, the equation of the image is \( y = -f(x) \). So, an equation of the image is:

\[
\begin{align*}
f(x) &= -(x - 2)^3 - 4 \\
f(x) &= -(x - 2)^3 + 4
\end{align*}
\]

4.3

5. Here is the graph of \( y = h(x) \). Sketch the graph of each function below. Write the domain and range of each transformation image.

a) \( y = h(3x) \)

The graph of \( y = h(x) \) is compressed horizontally by a factor of \( \frac{1}{3} \).

For each point at the ends of the line segments on \( y = h(x) \), divide the \( x \)-coordinate by 3, plot the new points then join them for the graph of \( y = h(3x) \).

The domain is: \(-1 \leq x \leq 2\)

The range is: \(-2 \leq y \leq 3\)

b) \( y = \frac{1}{2} h(x) \)

The graph of \( y = h(x) \) is compressed vertically by a factor of \( \frac{1}{2} \).

For each point at the ends of the line segments on \( y = h(x) \), divide the \( y \)-coordinate by 2, plot the new points then join them for the graph of \( y = \frac{1}{2} h(x) \).

The domain is: \(-5 \leq x \leq 1\)

The range is: \(-1 \leq y \leq 1.5\)

c) \( y = 2h(-3x) \)

The graph of \( y = h(x) \) is stretched vertically by a factor of 2, compressed horizontally by a factor of \( \frac{1}{3} \), then reflected in the \( y \)-axis.

Use: \((x, y)\) on \( y = h(x) \) corresponds to \( \left(-\frac{x}{3}, 2y\right)\) on \( y = 2h(-3x) \)

<table>
<thead>
<tr>
<th>Point on ( y = h(x) )</th>
<th>Point on ( y = 2h(-3x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3, -2))</td>
<td>((1, -4))</td>
</tr>
<tr>
<td>((0, 3))</td>
<td>((0, 6))</td>
</tr>
<tr>
<td>((3, 2))</td>
<td>((-1, 4))</td>
</tr>
<tr>
<td>((6, 2))</td>
<td>((-2, 4))</td>
</tr>
</tbody>
</table>

Plot the points, then join them.

The domain is: \(-2 \leq x \leq 1\)

The range is: \(-4 \leq y \leq 6\)
6. The graph of \( y = g(x) \) is the image of the graph of \( y = f(x) \) after a transformation. Corresponding points are labelled. Write an equation of the image graph in terms of the function \( f \).

The graph has not been translated, so an equation of the image graph has the form: \( y = af(bx) \).

A point \((x, y)\) on \( y = f(x) \) corresponds to the point \((\frac{x}{b}, ay)\) on \( y = af(bx) \).

The image of \( A(1, 1) \) is \((\frac{1}{b}, 1a)\), which is \( A'(−8, 4) \).

Compare coordinates: \( b = \frac{1}{8} \) and \( a = 4 \).

An equation of the image graph is: \( y = 4f(\frac{1}{8}x) \).

4.4

7. Here is the graph of \( y = f(x) \). On the same grid, sketch the graph of \( y - 4 = 3f(2(x-5)) \). Write the domain and range of the transformation image.

Compare: \( y - k = af(b(x-h)) \) to \( y - 4 = 3f(2(x-5)) \)

\( k = 4, a = 3, b = 2, \) and \( h = 5 \)

A point \((x, y)\) on the graph of \( y = f(x) \) corresponds to the point \( \left(\frac{x}{2} + 5, 3y + 4\right) \) on the graph of \( y - 4 = 3f(2(x-5)) \).

<table>
<thead>
<tr>
<th>Point on ( y = f(x) )</th>
<th>Point on ( y - 4 = 3f(2(x-5)) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, -4))</td>
<td>((5, -8))</td>
</tr>
<tr>
<td>((1, -3))</td>
<td>((5.5, -5))</td>
</tr>
<tr>
<td>((4, -2))</td>
<td>((7, -2))</td>
</tr>
<tr>
<td>((9, -1))</td>
<td>((9.5, 1))</td>
</tr>
</tbody>
</table>

Plot the points, then join them.
From the graph of \( y - 4 = 3f(2(x-5)) \), the domain is: \( x \geq 5 \); and the range is: \( y \geq -8 \).
8. The graph of \( y = g(x) \) is the image of the graph of \( y = f(x) \) after a combination of transformations. Corresponding points are labelled. Write an equation of the image graph in terms of the function \( f \). Justify your answer.

The equation of the image graph can be written as: \( y - k = af(b(x - h)) \)

The horizontal distance between A and B is 2.
The vertical distance between A and B is 4.
The horizontal distance between A’ and B’ is 2.
The vertical distance between A’ and B’ is 2.
The graph of \( y = f(x) \) has been compressed vertically by a factor of \( \frac{1}{2} \) and reflected in the x-axis, so \( a = -\frac{1}{2} \).
There is no horizontal stretch or compression, so \( b = 1 \).
Since B(4, 0) lies on the x-axis, it will not move after the vertical compression and reflection.
Determine the translation that would move B(4, 0) to B’(2, −3).
A translation of 2 units left and 3 units down is required, so \( h = -2 \) and \( k = -3 \).

An equation for the image graph is: \( y + 3 = -\frac{1}{2}f(x + 2) \)

9. The point (2, 2) lies on the graph of \( y = \frac{1}{4}x^3 \). After a combination of transformations, the equation of the image graph is

\[
y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right) \]

What are the coordinates of the point that is the image of (2, 2)?

Compare: \( y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right) \) with \( y - k = af(b(x - h)) \):

\[
k = -6, \ a = 5, \ b = 2, \ \text{and} \ h = 3
\]

A point \((x, y)\) on the graph of \( y = \frac{1}{4}x^3 \) corresponds to the point \((\frac{x}{2} + 3, 5y - 6)\) on the graph of \( y + 6 = 5\left(\frac{1}{4}(2(x - 3))^3\right) \).

Substitute \( x = 2 \) and \( y = 2 \) in the expression for the coordinates above.

\[
\left(\frac{2}{2} + 3, 5(2) - 6\right) = (4, 4)
\]

The image of (2, 2) has coordinates (4, 4).
10. Determine the inverse of each function, then sketch graphs of the function and its inverse.

a) \( y = -\frac{2}{5}x + 3 \)

Write: \( x = -\frac{2}{5}y + 3 \)
Solve for \( y \).
\[
5x = -2y + 15
2y = -5x + 15
y = \frac{-5x + 15}{2}
\]
The graph of \( y = -\frac{2}{5}x + 3 \) is a line with \( y \)-intercept 3 and slope \( -\frac{2}{5} \).
Reflect points on the graph of \( y = -\frac{2}{5}x + 3 \) in the line \( y = x \).
Join the points for the graph of \( y = \frac{-5x + 15}{2} \).

b) \( y = (x - 3)^2 + 7 \)

Write: \( x = (y - 3)^2 + 7 \)
Solve for \( y \).
\[
(y - 3)^2 = x - 7
y - 3 = \pm \sqrt{x - 7}
y = \pm \sqrt{x - 7} + 3
\]
The graph of \( y = (x - 3)^2 + 7 \) is the image of the graph of \( y = x^2 \) after a translation of 3 units right and 7 units up.
Reflect points on the graph of \( y = (x - 3)^2 + 7 \) in the line \( y = x \).
Join the points for the graph of \( y = \pm \sqrt{x - 7} + 3 \).

11. Restrict the domain of the function \( y = f(x) \) so its inverse is a function.

Sample response: Sketch the graph of the inverse by reflecting points in the line \( y = x \).
The inverse is a function if the domain of \( y = f(x) \) is restricted to \( x \leq 3 \) or \( x \geq 3 \).
12. A graph was reflected in the line $y = x$.
Its reflection image $y = g(x)$ is shown.
Determine an equation of the original graph in terms of $x$ and $y$.
Justify your answer.

a)

Use the line $y = x$ to sketch the graph of the inverse. This line has $y$-intercept 5, and slope $-4$.
So, its equation is: $y = -4x + 5$

b)

Use the line $y = x$ to sketch the graph of the inverse. This curve is a parabola that has vertex $(0, 3)$, and is congruent to $y = -x^2$.
So, its equation is: $y = -x^2 + 3$