2.1

1. Sketch a graph of each absolute function. Identify the intercepts, domain, and range.

a) \( y = | -4x + 2 | \)

Draw the graph of \( y = -4x + 2 \).
It has \( x \)-intercept 0.5.
Reflect, in the \( x \)-axis, the part of the graph that is below the \( x \)-axis.
From the graph, the \( x \)-intercept is 0.5, the \( y \)-intercept is 2, the domain of \( y = | -4x + 2 | \) is \( x \in \mathbb{R} \), and the range is \( y \geq 0 \).

b) \( y = | (x + 4)(x - 2) | \)

Draw the graph of \( y = (x + 4)(x - 2) \).
It has \( x \)-intercepts -4 and 2.
The axis of symmetry is \( x = -1 \), so the vertex is at \( (-1, -9) \).
Reflect, in the \( x \)-axis, the part of the graph that is below the \( x \)-axis.
From the graph, the \( x \)-intercepts are -4 and 2, the \( y \)-intercept is 8, the domain of \( y = |(x + 4)(x - 2)| \) is \( x \in \mathbb{R} \), and the range is \( y \geq 0 \).

2.2

2. Identify the equation of the vertical asymptote of the graph of \( y = \frac{1}{-2x + 5} \), then graph the function.

The graph of \( y = -2x + 5 \) has slope -2, \( x \)-intercept \( \frac{5}{2} \), and \( y \)-intercept 5.
The graph of \( y = \frac{1}{-2x + 5} \) has a horizontal asymptote \( y = 0 \) and a vertical asymptote \( x = \frac{5}{2} \).
Points (3, -1) and (2, 1) are common to both graphs. Some points on \( y = -2x + 5 \) are (1, 3), (0, 5), (4, -3), and (5, -5).
So, points on \( y = \frac{1}{-2x + 5} \) are \( (1, \frac{1}{3}) \), (0, 0.2), \( (4, -\frac{1}{3}) \), and (5, -0.2).
3. Use the graph of \( y = f(x) \) to sketch a graph of \( y = \frac{1}{f(x)} \).

Identify the equations of the asymptotes of the graph of each reciprocal function.

a)

Horizontal asymptote: \( y = 0 \)

\( x \)-intercept is \( \frac{1}{2} \), so vertical asymptote is \( x = \frac{1}{2} \).

Points (0, 1) and (1, –1) are common to both graphs.

Some points on \( y = f(x) \) are: (–1, 3) and (2, –3).

So, points on \( y = \frac{1}{f(x)} \) are \((-1, \frac{1}{3})\) and \((2, -\frac{1}{3})\).

b)

Horizontal asymptote: \( y = 0 \)

\( x \)-intercept is 6, so vertical asymptote is \( x = 6 \).

Points (7, 1) and (5, –1) are common to both graphs.

Some points on \( y = f(x) \) are: (8, 2) and (4, –2).

So, points on \( y = \frac{1}{f(x)} \) are (8, 0.5) and (4, –0.5).
4. Use graphing technology. For which values of $q$ does the graph of

$$y = \frac{1}{-(x - 3)^2 + q}$$

have:

a) no vertical asymptotes? Explain.

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have no vertical asymptotes, the quadratic function must have no $x$-intercepts. So, the vertex of the quadratic function must be below the $x$-axis; that is, $q < 0$.

b) one vertical asymptote? Explain.

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have one vertical asymptote, the quadratic function must have one $x$-intercept. So, the vertex of the quadratic function must be on the $x$-axis; that is, $q = 0$.

c) two vertical asymptotes? Explain.

Look at the quadratic function $y = -(x - 3)^2 + q$. The graph opens down. For the graph of its reciprocal function to have two vertical asymptotes, the quadratic function must have two $x$-intercepts. So, the vertex of the quadratic function must be above the $x$-axis; that is, $q > 0$.

5. Determine the equations of the vertical asymptotes of the graph of each reciprocal function.

Graph to check the equations.

a) $y = \frac{1}{(x - 2)^2 - 9}$

$(x - 2)^2 - 9 = 0$ when $(x - 2)^2 = 9$; that is, when $x = 5$ or $x = -1$. So, the graph of $y = \frac{1}{(x - 2)^2 - 9}$ has 2 vertical asymptotes, $x = 5$ and $x = -1$. I used graphing technology to show that my equations are correct.

b) $y = \frac{1}{-(x - 2)^2 - 9}$

$-(x - 2)^2 - 9 = 0$ when $(x - 2)^2 = -9$. Since the square of a number is never negative, the graph of $y = -(x - 2)^2 - 9$ has no $x$-intercepts, and the graph of $y = \frac{1}{-(x - 2)^2 - 9}$ has no vertical asymptotes. I used graphing technology to show that my equations are correct.
6. Graph each reciprocal function. Identify the vertical asymptotes, if they exist. Explain your work.

a) \( y = \frac{1}{-2(x - 3)^2 - 2} \)

Graph \( y = -2(x - 3)^2 - 2 \).
The graph opens down, has vertex \((3, -2)\), and has no \(x\)-intercepts.
So, the graph of \( y = \frac{1}{-2(x - 3)^2 - 2} \) has no vertical asymptote and has Shape 1.
Horizontal asymptote: \( y = 0 \)
Points \((2, -4)\), \((3, -2)\), and \((4, -4)\) lie on \( y = -2(x - 3)^2 - 2 \), so points \((2, -0.25), \left(\frac{1}{3}, -0.5\right)\), and \((4, -0.25)\) lie on \( y = \frac{1}{-2(x - 3)^2 - 2} \).
Join these points with a smooth curve that approaches the horizontal asymptote.

b) \( y = \frac{1}{(x - 5)^2} \)

Graph \( y = (x - 5)^2 \).
The graph opens up, has vertex \((5, 0)\), and has 1 \(x\)-intercept.
So, the graph of \( y = \frac{1}{(x - 5)^2} \) has 1 vertical asymptote, \( x = 5 \), and has Shape 2.
Horizontal asymptote: \( y = 0 \)
Plot points where the line \( y = 1 \) intersects the graph of \( y = (x - 5)^2 \). These points are common to both graphs.
Through these points, draw 2 smooth curves that approach the asymptotes.