Lesson 2.3 Math Lab: Assess Your Understanding, pages 112–114

1. Without graphing, predict the number of vertical asymptotes of the graph of each reciprocal function. Identify the equation of each asymptote.
   a) \( y = \frac{1}{(x + 2)(x - 4)} \)

   The \( x \)-intercepts of the related quadratic function are \(-2\) and \(4\).
   There are 2 vertical asymptotes: \( x = -2 \) and \( x = 4 \)

   b) \( y = \frac{1}{(-3x + 1)^2} \)

   The \( x \)-intercept of the related quadratic function is \( \frac{1}{3} \).
   There is 1 vertical asymptote: \( x = \frac{1}{3} \)

   c) \( y = \frac{1}{x^2} \)

   The \( x \)-intercept of the related quadratic function is \(0\).
   There is 1 vertical asymptote: \( x = 0 \)

   d) \( y = \frac{1}{4x^2 + 3} \)

   The related quadratic function has no \( x \)-intercepts.
   There are no vertical asymptotes.

2. Look at your answers to question 1. When the equation of a reciprocal quadratic function is given in factored form, how can you tell how many vertical asymptotes its graph will have?

   To tell how many vertical asymptotes the graph of a reciprocal quadratic function will have, I look at the expression in the denominator.
   When the expression cannot be factored, there are no vertical asymptotes.
   When the expression has two identical factors, there is 1 vertical asymptote.
   When the expression has two different factors, there are 2 vertical asymptotes.
3. Graph \( y = a(x - p)^2 + q \) and \( y = \frac{1}{a(x - p)^2 + q} \) on the same screen for 6 different sets of values of \( a, p, \) and \( q \). Sketch what you see on the screen. How can you use the signs of \( a, p, \) and \( q \) to determine the number of vertical asymptotes of the graph of the function \( y = \frac{1}{a(x - p)^2 + q} \)?

When \( a \) is negative, the graph opens down:
If \( q \) is also negative, the related quadratic function has no \( x \)-intercepts, so there are no vertical asymptotes. For example:
\[
y = -2(x - 3)^2 - 1 \quad \text{and} \quad y = \frac{1}{-2(x - 3)^2 - 1}
\]

If \( q \) is positive, the related quadratic function has 2 \( x \)-intercepts, so there are 2 vertical asymptotes. For example:
\[
y = -2(x + 3)^2 + 1 \quad \text{and} \quad y = \frac{1}{-2(x + 3)^2 + 1}
\]

If \( q = 0 \), the related quadratic function has 1 \( x \)-intercept, so there is 1 vertical asymptote. For example:
\[
y = -2(x - 3)^2 \quad \text{and} \quad y = \frac{1}{-2(x - 3)^2}
\]

When \( a \) is positive, the graph opens up:
If \( q \) is also positive, the related quadratic function has no \( x \)-intercepts, so there are no vertical asymptotes. For example:
\[
y = 2(x + 3)^2 + 1 \quad \text{and} \quad y = \frac{1}{2(x + 3)^2 + 1}
\]

If \( q \) is negative, the related quadratic function has 2 \( x \)-intercepts, so there are 2 vertical asymptotes. For example:
\[
y = 2x^2 - 1 \quad \text{and} \quad y = \frac{1}{2x^2 - 1}
\]

If \( q = 0 \), the related quadratic function has 1 \( x \)-intercept, so there is 1 vertical asymptote. For example:
\[
y = 2(x - 3)^2 \quad \text{and} \quad y = \frac{1}{2(x - 3)^2}
\]
4. Predict the vertical asymptotes of the graph of each reciprocal function. Justify your prediction. Graph to check your predictions.

a) \( y = \frac{1}{(x+1)^2 - 9} \)

Since the value of \( a \) is positive and the value of \( q \) is negative, I predict the graph of the reciprocal function will have 2 vertical asymptotes.

\[ y = \frac{1}{(x+1)^2 - 9} \text{ is undefined when} \]

\[ (x+1)^2 - 9 = 0 \]

\[ (x+1)^2 = 9 \]

\[ x + 1 = 3 \text{ or } x + 1 = -3 \]

\[ x = 2 \quad x = -4 \]

So, the lines \( x = 2 \) and \( x = -4 \) are vertical asymptotes.
The graph shows my prediction is correct.

b) \( y = \frac{1}{(x+1)^2} \)

Since the value of \( a \) is positive and the value of \( q \) is 0, I predict the graph of the reciprocal function will have 1 vertical asymptote.

\[ y = \frac{1}{(x+1)^2} \text{ is undefined when} \]

\[ (x+1)^2 = 0 \]

\[ x + 1 = 0 \]

\[ x = -1 \]

So, the line \( x = -1 \) is a vertical asymptote.
The graph shows my prediction is correct.

c) \( y = \frac{1}{-(x+1)^2 + 16} \)

Since the value of \( a \) is negative and the value of \( q \) is positive, I predict the graph of the reciprocal function will have 2 vertical asymptotes.

\[ y = \frac{1}{-(x+1)^2 + 16} \text{ is undefined when} \]

\[ -(x+1)^2 + 16 = 0 \]

\[ -(x+1)^2 = -16 \]

\[ (x+1)^2 = 16 \]

\[ x + 1 = 4 \text{ or } x + 1 = -4 \]

\[ x = 3 \quad x = -5 \]

So, the lines \( x = 3 \) and \( x = -5 \) are vertical asymptotes.
The graph shows my prediction is correct.